NUMERACY

A Common Essential Learning for
Saskatchewan Students

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PREFACE

In 1986, after a thorough review of educational goals for Saskatchewan schools was completed, the Department of Education published its findings and recommendations in the report Program Policy Proposals. A key recommendation was to incorporate the following six categories of common essential learnings in all courses of study offered in Saskatchewan schools: communication skills, creative and critical thinking, independent learning skills, numerical and quantitative literacy, personal and social skills and values, and technological literacy.

Each category includes those skills, attitudes, and processes necessary for functioning in society no matter what students choose to do after completing their elementary and secondary school education. A major responsibility of schools will be to ensure that all graduates are proficient in these common essential learnings regardless of their programs of study.

The report did not attempt to provide an extensive list of these common essential learnings. Instead, all categories of learnings were defined only broadly. For example, quantitative estimation, calculation, and statistical analysis were the only aspects of numerical and quantitative literacy mentioned in the report. The report recommended that a more detailed elaboration of each category be developed by the Department of Education in conjunction with experts in the related fields of education.

This report will focus on the common essential learnings comprising numerical or quantitative literacy, hereafter referred to as numeracy.

I. DEFINING NUMERACY

> The president's command of facts is questioned during a press conference. The Reagan impersonator responds, "If you're right 90 percent of the time, why quibble about the remaining three percent." (Line from the satirical play Rap Master Ronnie)

Undoubtedly, there is general agreement that every adult needs to know some mathematics to function effectively in our society where quantification abounds. Helping children develop this knowledge is considered to be one of the most important goals of education. The value of mathematics instruction was recognized in a recent United States Gallup survey that found, for the first time in its history of polling public opinions, mathematics, and not English, was considered the most important school subject.

The teaching of mathematics receives widespread public support because of its usefulness. This usefulness is perceived in different ways: as arithmetic skills needed at home and work, as mathematical techniques useful in science and modern technology, and as methods for decision making in commerce, industry and government.

These differing perceptions of the uses of mathematics influence what are thought to be the goals of elementary and secondary school mathematics instruction. While everyone agrees that mathematics is important to learn, there is often little agreement about the amount or type of mathematics to be learned. Consequently, it is difficult and perhaps hazardous to take on the task of identifying the mathematical skills and knowledge essential for all students regardless of their abilities, interests, and ultimate career choices. Any list of essential learnings is bound to be criticized by some for omitting "important" skills and by others for including "unneeded" ones. Notwithstanding these obvious difficulties, this paper will attempt to identify those mathematical ideas, understanding and skills whose mastery is essential for living a productive adult life.

Numeracy is not a neological whim of this author, but a term recognized in the 1976 supplement to the Oxford English Dictionary. The origin of the term can be traced to a 1959 British publication that expressed a concern for the inability of scientists and humanists to communicate with each other. This gap in communication was predicted to widen unless scientists became more literate and humanists became more numerate.

The words "numeracy" and "numerate" are beginning to appear more frequently in the mathematics education literature, but their meanings have changed considerably over the past 25 years. In the original British report, numeracy denoted a quite sophisticated knowledge of mathematics and science as the following excerpt indicates:

> On the one hand . . . an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification. On the other hand . . . the need in the modern world to think quantitatively, to realize how far our problems are problems of degree even when they appear as problems of kind. (Committee of Inquiry into the Teaching of Mathematics in Schools [Cockcroft Report], 1983, p. 11)

This association with science has disappeared and most writers use the terms to indicate a much lower level of mathematical understanding than was intended originally. Recent dictionary definitions reflect these changes in meaning. The Oxford English Dictionary defines numeracy as "an ability with or knowledge of numbers," while the Collins Concise Dictionary provides the narrower version, "the ability to perform basic arithmetic operations."

Neither of these dictionary definitions provides an adequate picture of numeracy. "An ability with or knowledge of numbers" does not specify what sort of ability or what level of knowledge constitutes a mastery of numbers. Consequently, this definition gives little help in distinguishing between numerate and innumerate people.

"The ability to perform basic operations" is more specific but still inadequate because the ability to compute is of no value until it is applied to the solution of a problem. A person knowing how to compute but not knowing when to compute should not be considered numerate. Moreover, for problems requiring some tedious calculations, numerate adults often use calculators rather than the pencil-and-paper methods they were taught in school.

The Committee of Inquiry into the Teaching of Mathematics in Schools (hereafter referred to by the chairman's name, Cockcroft) believed that functionality is an important characteristic of numeracy and numerate people are seen as possessing these two attributes (1983, p. 11):

1. An "at-homeness" with numbers and an ability to make use of mathematical skills that enables an individual to cope with the practical demands of everyday life.
2. An appreciation and understanding of information that is presented mathematically, for instance, in graphs, charts and tables or by reference to percentage increase or decrease.

The following definitions of numeracy and numerate have guided the writing of this report.

> **Numeracy can be considered a complex aggregrate of knowledge, skills and appreciations needed to analyze, interpret, and use quantitative information intelligently and to solve problems encountered in adult life. Numerate people are those who can cope confidently and competently with most everyday situations demanding the use of mathematical ideas and, when necessary, can learn new techniques and applications within a reasonably short period of time. In short, being numerate means knowing how to use**

> **mathematical ideas to carry out routine tasks, to solve realistic problems, and to interpret commonly seen quantitative statements.**

If schools are expected to do their part in ensuring that students are sufficiently numerate to handle the tasks of adult life, as wise consumers, as productive workers and as responsible citizens active in community affairs, information about the nature of these tasks must be gathered and provided to teachers, administrators and curriculum developers. Such information can be helpful in evaluating the effectiveness of school programs in aiding children to handle the commonplace mathematical tasks of the adult world. Parts 2 and 3 of this report examine the numeracy demands of everyday adult life, both inside and outside the workplace.

II. NUMERACY DEMANDS OF LIFE OUTSIDE THE WORKPLACE

Hardly a day in most of our lives passes by where some form of mathematical idea is not used. Daily, we carry out a number of tasks routinely, counting objects, reading clocks, counting money, reading prices and finding the page location of a book or magazine. These skills are usually highly automatized and performed effortlessly with little conscious thought. Although only number skills taught in the elementary school are mainly needed, many people struggle occasionally with non-routine tasks such as completing an annual tax return or estimating the amount and cost of new carpeting. In these exceptional cases, many people seek outside help.

Many lists of "essential" mathematics supposedly necessary for carrying out routine tasks have been compiled. These lists, in turn, are often used in the development of courses preparing students for the tasks of daily life. For example, California has prepared a guide listing proficiencies and performance indicators for use in designing and revising curricula dealing with essential living skills in consumer and homemaking education. Using a variety of published lists, Cowan and Clary (1978) prepared a composite list of essential basic mathematical skills needed to function as effectively as possible in everyday life, including vocational pursuits. An attempt was made to "validate" the list of 54 skills by asking a relatively large sample of professional educators, businesspersons, and lay people to indicate whether all, most, half, few or none of the students in their geographical area should be expected to know a listed skill. The results of the returned questionnaires were used to prepare an ordered list of essential mathematical skills. Similar lists of essential life-skills have been prepared in many Canadian provinces, including Saskatchewan. Other than by surveying opinions, rarely is any further attempt made to validate these lists.

The validity of these lists of "essential mathematical skills" can be seriously questioned because the few studies that have examined how people actually use mathematical ideas demonstrate that many notions claimed to be essential are not at all essential for functioning in the everyday world of the consumer and citizen. There is compelling evidence that the bulk of mathematics taught in school is rarely used by the great majority of people.

Surveys of Everyday Numeracy

A British study sponsored by the Advisory Council for Adult and Continuing Education (1982) - hereafter referred to as the ACACE study - investigated the use of mathematics in daily life by interviewing 107 adults who varied in sex, age, educational background and occupation. Each participant's proficiency in basic mathematics was also assessed informally.

To tease out some of the number knowledge and skills that each person used regularly, the ACACE study presented various quantitative situations likely to be familiar to adults. Probing questions such as the following were used to encourage the adults to discuss their typical behaviour: (1) In the supermarket, how do you know that you have enough to pay for the items in your trolley (shopping cart)?; (2) In a pub (fish and chip shop, restaurant), do you work out the cost of what you want before you order?; (3) When you cook, do you weigh the ingredients? If not, do you weigh anything?; (4) When shopping for one or two items, do you check the change?; (5) If you see something you want in the shops and it's marked "10% off," do you work out how much you would save? (If no) Could you, if you wanted?

Those interviewed varied considerably in their willingness and ability to handle the common problems used in the ACACE study. For example, only about 40% said they worked out prices before ordering in a pub, fish and chip shop, or restaurant. One reply with a ring of truth given by a pub dweller was "Well, I add it accurately at the start of the evening." Most of those interviewed said they regularly checked change and occasionally measured materials or weighed objects.

About one third of the sample thought that all percentages were meaningless, including straightforward ones like 10%. "They're beyond me," You've got me there," and "I'm completely lost with percentages" were typical replies.

Interestingly, although about 70% of these adults claimed that they had access to a calculator, one third said they never used it. Those who did find the calculator useful used it to operate on awkward numbers: comparing prices in bulk shopping, checking mortgage payments and income tax, estimating petrol consumption.

There appeared to be no relationship between the extent to which adults use mathematics and the extent of their initial education. In the ACACE study, there were people with science degrees who claimed to use no arithmetic and those with no qualifications who displayed a high level of mathematical competence.

About half of the adults involved in the ACACE study were interviewed again at greater length. They were questioned on a wide range of everyday situations requiring straightforward mathematics. Some questions required calculations, some required only an explanation of the method, and some required the interpretation of quantitative information. Original documents such as bills, pay-slips and timetables were often used. For example, the interviewees were shown an actual printed wage slip for a person named Jennifer that contained about 25 labelled boxes. Printed on the wage slip were details of Jennifer's pay and deductions and the participants were asked to determine the total deductions from her pay.

No questions in the study required computation unless they were tied to a real problem situation. All participants were free to use any calculation method or device they wished including mental, pencil and paper, or calculator.

These adults generally had little difficulty in producing reasonable answers to most of the questions. However, they frequently had difficulty in handling problems involving percentages, ratios, graphs and charts, timetables, metric units and metric/imperial conversions. More than half could not do the following tasks successfully: use a timetable to determine the arrival time of a departing train; compare prices of gasoline selling in litres or gallons; provide a reasonable estimate for the amount of sugar needed to make jam using 3 lbs. of strawberries if a recipe called for 3 1/2 lbs. sugar for every 4 lbs. of strawberries; calculate gasoline consumption given the initial odometer and gas gauge readings, two "fill-up" quantities, and the final odometer and gas gauge readings.

Less than one third gave a reasonable answer to a question asking for the effect on prices when the inflation rate fell from 17.4% to 17.2% to finally 12%. Instead of viewing this change as a lessening of the rate of price increases, most of the adults thought a corresponding drop in prices should accompany a fall in the rate of inflation. The ACACE enquiry officer conducting this study was particularly distressed by this "alarmingly widespread (and confident) misunderstanding of inflation revealing a lack of comprehension of the most commonly quoted feature of the national economy. The mathematical aspect of the question on inflation - referring to a change of rate, like easing off on the accelerator - scarcely entered into the replies" (p. 56).

To determine if the difficulties encountered by the ACACE sample were representative of the British population, 10 questions of the kind that had been used in the interviews were included in a Gallup survey of 3000 adults. The results of the larger survey indicated that the findings of the smaller ACACE study were in no way atypical. For example, only 40% of the Gallup sample knew prices would still rise if inflation dropped from 20% to 15%.

The ACACE study also revealed that much of the computation laboriously practised in school was either forgotten or its relevance to everyday problem solving was never perceived. To compensate for the absence of a school-like procedure, many adults had developed other useful computational alternatives: for example, computing differences by mentally adding on instead of subtracting. The methods relating to mental arithmetic were clearly useful and important to most adults.

In contrast to the traditional notion fostered in schools of a single best or mathematically superior method for solving problems, there was considerable variation in the methods adults used to solve problems. There was a surprising reluctance, particularly by women, to estimate. Most adults seemed to assume that only exact answers were acceptable - even when approximate answers were called for. This reluctance to estimate, even when it is appropriate to do so, has been noted in other studies (see Campbell, 1983). Perhaps many adults view estimates as "second-rate" answers or confuse estimating with haphazard guessing.

Many adults were either unable or unwilling to use a calculator. Some who tried to use a calculator had been discouraged by the large number of extraneous digits to the right of the decimal point - for instance, when dividing by 3 - and had lacked confidence to persevere and to discover how to interpret the answers they had obtained. Others refused to use a calculator because they feared it would somehow make them "lazy." There were some adults, however, whose computational skills were weak and the use of the calculator enabled them to function confidently and competently.

Perhaps the most striking feature of the ACACE study was the feelings of anxiety, helplessness, fear, and, in some cases, guilt reported by many adults when they were asked to perform some seemingly straightforward number skill. The large number of adults who were asked originally but refused to participate in the study were suspected by the researchers to be similarly apprehensive about situations requiring the use of numbers: "Both direct and indirect approaches were tried, the word 'mathematics' was replaced by 'arithmetic' or 'everyday use of numbers', but it was clear in many cases that the reason for people's refusal to be interviewed was simply that the subject was mathematics" (p. 32).

Ironically, those adults in the ACACE study with the highest academic qualifications were often the most anxious and guilt-ridden because they thought they "ought" to have a confident understanding of mathematics even though this was not the case. Their anxiety was often compounded by an awareness that others expected them to be very numerate as they had been generally successful in other academic pursuits. As one subject commented, "People assume you're good at maths if you're good at other things."

While claiming they managed to function satisfactorily in everyday situations, some adults appeared to avoid the use of numbers whenever possible. Often these adults compensated for their lack of numeracy by employing a variety of coping strategies: for example, enlisting others to complete forms and read numbers. A thirty-year-old adult participating in a recent study (Hope & Madden, 1986) is representative of many innumerate adults who devise a variety of ways to cloak their inability to use straightforward number skills.

Although he had completed a grade 11 occupational program offered by a secondary school in Manitoba, was expressive and had a well-developed vocabulary, this adult had problems with most elementary number skills: recalling basic number facts, subtracting numbers like 32 and 16, reading numbers like 1318, and determining the denominations needed to pay a bill of $38.16. To disguise his poor money skills, his usual procedure for going grocery shopping was to remove all bills smaller than $20.00 (he could recognize the values of paper currency) from his wallet before entering a store. When given the total by the cashier, he would reach into his wallet and pull out a "bunch" of bills and toss "them on the counter - with authority," hoping that there was sufficient money to cover his purchases. Unable to make use of the resulting change, he collected it in a large jar at home, and periodically took the change to a confectionary store during a quiet hour for an accomodating cashier or owner to count. He used a similar show of bravado at the bank where usually an obliging teller would complete his banking forms.

Like most innumerate adults, his clear lack of number skills had prevented him from applying for jobs or taking courses of training.

Needless to say, he was a very frustrated person:

> Sometimes, I feel like I was born in the wrong time. If I had been born in the eighteenth or nineteenth century, I probably would have done quite well. As it is, I feel like a fool when I see eight-year-old kids that can do simple things that I can't, no matter how hard I've tried. It's frustrating.

Capon and Kuhn (1979) tried to evaluate how well adults cope with everyday quantitative situations. They examined the abilities of 50 female shoppers to identify the better buy of two sizes of a common item sold in a supermarket. Shoppers were approached outside a supermarket and shown two bottles of garlic powder that varied both in price and size. The smaller 1.25 ounce bottle sold for 41 cents, while the larger 2.37 ounce bottle sold for 77 cents. Each adult was supplied with pencil and paper and asked, "How could you tell which one is the better buy?" This testing procedure was repeated using two cans of deodorant, one marked 8 ounces and priced at $1.36 and the other marked 12 ounces and priced at $2.11.

Capon and Kuhn categorized the responses and explanations of the shoppers in the order of mathematical sophistication with price per ounce calculations ranked as the most sophisticated. About one half of the adults did not use proportional reasoning to determine the better buy. Approximately 30% used extraneous information typified by the following comments: "I always buy the larger size," "I don't like to shop often," or "The big one must be cheaper" (p. 450-451).

Because of the shoppers' responses and their general unwillingness to use proportional reasoning, Capon and Kuhn concluded that "formal operational reasoning among adults is far from universal even under these favorable assessment conditions" (p. 451). To the question whether the variability in the study was of serious practical significance, especially when the now common practice of unit pricing might compensate for an adult's lack of reasoning, Capon and Kuhn answer:

> Unless individuals have the ability to know for themselves how to use the kind of cognitive strategy investigated here, control of the consumer's purchasing behaviour remains to some significant degree in the hands of others. Thus, why adults have not mastered such strategies and how such mastery might be facilitated seem topics worthy of our concern. (p. 452)

The findings of these studies seem to suggest that many adults either avoid or cannot handle everyday quantitative situations. The validity of these findings can be questioned, however. Interviews, laboratory experimentations and even simulations conducted in researcher controlled environments may not be satisfactory replications of problem solving in the real world. Observing people actually engaged in everyday problem-solving activities is likely the only satisfactory method for gathering this type of information.

Recently, a number of studies have investigated the arithmetic practices of people as they function in naturally occurring quantitative situations. Murtaugh (1985) studied American grocery shoppers as they went about the business of shopping; De la Rocha (1985) studied dieters as they prepared their meals in the kitchen; Scribner (1985) studied commercial dairy workers as they went about their work; Carraher, Carraher, & Schliemann (1985) and Carraher & Schliemann (1985) went as customers to Brazilian open air markets to study the ability of young children to handle the arithmetic when unusual quantities of food were bought. All of the participants in these studies were later tested on arithmetic problems and procedures that were supposedly identical to those encountered in the out-of-school world.

To abbreviate the review of these studies, only Murtaugh's study will be reviewed in some detail. Little generalization will be lost because his methodology and findings are similar to the other studies, despite the great variation in settings and sample characteristics. The findings of all studies will then be presented as a whole.

Murtaugh examined the arithmetic procedures people use when shopping by following 24 people around supermarkets while observing and recording their problem-solving behavior. One of the purposes of the study was to identify the kinds of problems people ordinarily encounter in the course of grocery shopping that require the manipulation of quantitative information. Murtaugh was also interested in the relationship between problem formation and problem solving. Unlike the problems used by the ACACE researchers and Capon & Kuhn in which the problems were formulated and posed by the researchers, in Murtaugh's naturalistic study, shoppers had to generate their own problems and the way these problems are generated may largely determine how they go onto solving them.

Murtaugh found that, with the exception of one person, all shoppers made some use of arithmetic while grocery shopping. Eighty five percent of the arithmetic used by this group involved price comparisons. Most of these comparisons were relatively straightforward: e.g. comparing prices of two products of equal size or quantity.

The most complex supermarket arithmetic involved best buy comparisons where items were packaged in different amounts or sizes: about 30 percent of all supermarket calculations involved these type of comparisons. Although unit prices were provided by the supermarkets, shoppers in the study generally ignored them. Rarely did a shopper calculate unit prices to make comparisons. Instead, two procedures were frequently used.

When the ratios of the compared quantities were 2:1 or 3:1, prices were compared by doubling or tripling the price of the smaller size. This method is not only easier to do mentally than unit price calculations, but it also provides a shopper with information about the amount saved. For example, one shopper compared two boxes of sugar, one priced at $2.16 for five pounds, the other at $4.30 for ten pounds. The shopper explained: "The five pounds would be four dollars and 32 cents, versus four dollars and 30 cents. I guess I'm going to have to buy the ten-pound bag just to save a few pennies" (p. 190). In this case, unit pricing would have revealed that the larger size cost 0.2 cents less per pound than the smaller size - meaningless information for most shoppers.

Even where the size ratio was not exactly 2:1, the shoppers used the doubling procedure effectively. For example, one shopper compared two cans of tomato sauce containing 15 ounces for 39 cents and 8 ounces for 18 cents. The shopper decided that two cans of the small size were the best deal by reasoning, "That's 36 cents for 16 ounces, and the bigger is 39 for 15" (p. 190). To this shopper, it was more relevant and convenient to consider relative price differences than to compute and compare two unit prices: in this case, comparing 39/15 = 2.6 cents and 18/8 = 2.25 cents.

Murtaugh disputes Capon and Kuhn's contention that the reasoning capacities of ordinary people are extremely limited. He points out that in the Capon and Kuhn study "better buy" was defined by the researchers as "lowest price per ounce" but, as Murtaugh demonstrated, price per ounce is not a particularly useful piece of information for shoppers. Murtaugh explains:

> Shoppers apparently feel it is not worth the effort to calculate the price per single ounce, when a single ounce is neither purchased nor consumed. In contrast, the procedures that shoppers do employ reflect concerns for both price and quantity, as well as the organization of inputs in the environment. (p. 192)

Based on the findings of these naturalistic studies, several conclusions can be made. First, people often use problem-solving procedures more complex than those that are taught in school. For example, Scribner found that dairy pre-loaders were doing arithmetic typical of 8th and 9th graders, although they averaged about 6 grades of schooling.

Second, people are accurate calculators when involved in a meaningful activity. The Brazilian children in the market produced correct arithmetic results 99 percent of the time. The dairy pre-loaders made no errors and Murtaugh's shoppers were correct 98 percent of the time in best-buy arithmetic. This contrasts significantly with school-like tests on seemingly comparable problems where the Brazilian children averaged 74 percent, the dairymen averaged 64 percent, and the shoppers averaged 59 percent (Lave, 1985, p. 173). Also, there is little relationship between performance on arithmetic tests (computation, basic fact recall, word problems, etc.) and performance in solving problems encountered in daily life.

Moreover, the techniques of computation that adults use to solve real-world problems vary from one situation to another. In the world of the adult, people tend to select techniques according to a number of criteria including the purpose of the calculation, the size and types of numbers, the degree of precision and accuracy needed and, most importantly, helpful contextual cues in the environment. For example, shoppers use a variety of ways to compare prices. Dieters use handy tools like measuring cups to calculate fractional amounts instead of resorting to pencil-and-paper arithmetic - if a recipe calling for 3/4 cup of milk is to be halfed, then pour in 3/4 cup of milk and pour back one half of the milk in the cup into the milk container. In other words, adults in real life use situationally specific problem-solving techniques rather than general procedures suitable for all situations.

In contrast to the flexibility of thinking exhibited by adults while performing everyday tasks, algorithmic thinking is usually resorted to when school-like computations and problems are confronted in testing situations, even when the procedures adults attempt to produce are either half-remembered or inappropriate. The adults in these naturalistic studies, like the ones in the ACACE study, apologized to the researchers for not being able to produce what they called "real math" - the procedures taught in school. Moreover, they tended to denigrate the arithmetic practices they used routinely to solve everyday problems, despite their usefulness and effectiveness.

In summary, there seems not to be a direct connection between problem solving in the real world and the arithmetic taught in school. Unlike the methods seen in everyday practice, the algorithmic procedures emphasised in school are done in the absence of a context. In daily life, a calculation is done to solve a self-defined problem; in schools, a calculation is stripped of meaning and done for its own sake. The researchers involved in the Adult Math Skills Project (Lave, de la Rocha, Faust, Murtaugh & Migalski, 1983) used the following analogy to highlight the importance of context in solving the problems of daily life:

> All aspects of the investigation reported here converge on the proposition that everyday life is NOT like a tossed salad, in which arithmetic is one ingredient, which like all others, keeps its separate identify and integrity, making co-appearances with other vegetables in the salad bowl. It is much more like a good bowl of chili, where each of the ingredients is transformed by association with the others - in the end barely recognizable, and tasting different. (p. 41)

III. NUMERACY DEMANDS OF LIFE INSIDE THE WORKPLACE

Once the distinct preserve of the physical and mathematical sciences, computers with their enormous number-crunching capabilities have entered the workplace forcing more and more workers to become familiar with numbers and statistics, especially the interpretation of quantitatively expressed information. The substantial decrease in the proportion of menial occupations has been followed by an increase in the proportion of jobs requiring a minimal level of competence in the skills of numeracy. Business managers, administrators and public servants need to be sufficiently numerate to interpret the data that are now routinely used for rational decisionmaking. According to some authorities, numeracy has become so essential in meeting the demands of some jobs that it looms now as the successor of literacy:

> Just as the possession of basic literacy once marked a significant dividing line in society between the powerful and powerless or between the priests and the people, so numeracy is now the basic skill that increasingly establishes a status boundary in the world of work. (Cohen, 1982, p. 5)

Outside the workplace, innumerate adults can manage successfully by relying on various coping strategies and on the goodwill of other more numerate persons; inside the workplace, their innumeracy becomes apparent and they face a bleak future of unemployment and low-paying, menial jobs. Numeracy is a "China wall" keeping the innumerate from entering a satisfying career.

Are students sufficiently numerate to enter the workforce and to complete training programs and apprenticeships successfully? Judging from surveys conducted in several Western countries, employers are generally dissatisfied with the numeracy levels of prospective young workers.

Youck (1974) cites the conclusions of two Canadian surveys of employers' perceptions of the mathematical literacy of graduating students:

> Metropolitan Life Insurance Company of Canada, which employs about 1000 people each year, has found that approximately 80 percent of the high school graduates who apply are not able to pass the fraction and percentage tests that the Company administers to all its applicants.
>
> In 1973, a survey was made of forty-six Regina industries with regard to educational requirements. All forty-six stated they were shocked at how weak the students they employ were in "The Three R's," especially in Arithmetic and Writing. (p. 1)

In 1980, the Western Australia Education Department surveyed a representative sample of employers in medium-to-large enterprises to assess their views on the effectiveness of schools in preparing young people for the workforce, with particular emphasis on literacy and numeracy. Twenty-two percent of the employers complained of the inability of many young people to perform simple arithmetic calculations, placing much of the blame on an apparent use of calculators in school. A somewhat exaggerated view of Australian students' inability to handle numbers was expressed by one employer: "To put it simply - while they [the students] are preoccupied in trying to work out 7 x 7, they are unable to grasp the things you are trying to teach them in relation to the job" (p. 19).

An Australian parliamentary committee reported that most briefs received from employers suggested that "too many school leavers [high school graduates] have to be rejected because they have insufficient mastery of literacy and numeracy for basic job requirements" (Senate Standing Committee on Education and the Arts, 1981, p. 3). While conceding that much of the complaint of employers was based on opinion surveys rather than on solid empirical evidence, the committee thought there was sufficient evidence to make this conclusion:

> . . . at least 20 percent of young people who leave Australian

> schools each year are insufficiently trained to compete effectively in a highly competitive job market or to survive the psychological traumas of being unemployed for lengthy periods. (p. 7)

One finding common to most studies of employer perceptions is that despite their often harsh criticism of the numeracy and literacy levels of young workers, employers report that suitable applicants are easy to find, especially in times of high unemployment. Employers' dissatisfaction with the effectiveness of schooling seems to be primarily based on the applications they receive rather than being a general criticism of the employees they eventually hire. In isolated areas such as mining towns in Queensland or northern Saskatchewan, however, there is not a large pool of workers to draw from. In these exceptional cases, employers complain that it is very difficult to find sufficiently literate and numerate workers.

Although employers present a one-sided and possibly uninformed view about the quality of the educational preparation of young workers, their views must be given careful consideration by educators. Moveover, there is some evidence from a variety of sources indicating that a significant minority of young people in Saskatchewan may be, at best, marginally numerate.

Giesbrecht (1977) evaluated Saskatchewan high school students' achievement of the 48 basic mathematical competencies outlined by the National Council of Teachers of Mathematics. These basic competencies are claimed to be those mathematically-related abilities needed to function as a citizen. Attainment of these competencies supposedly equips people to function adequately in the world of work and provides a foundation for entering into and functioning within that world (National Institute of Education, 1975, p. 149).

Giesbrecht administered two forms of the Beckmann-Beal Mathematical Competency Test for Enlightened Citizens to 851 grade 9, 856 grade 10, 816 grade 11 and 765 grade 12 students enrolled in a mathematics program in 161 Saskatchewan high schools. The Beckmann-Beal test is based upon the list of the 48 NCTM basic mathematical competencies. Giesbrecht used the criterion that a student had attained a competency when responding correctly to a question on Form A and its related counterpart on Form B.

He found the majority of students in grades 10, 11, and 12 attained a majority of the 48 NCTM competencies while the majority of grade 9 students did not. The proportions of students who did not attain one half of the competencies were 61% in grade 9 and 20% in grade 12. The proportions of students who did not attain one quarter of the competencies were 21% in grade 9 and 5% in grade 12.

Although most students could compute reasonably well, they experienced problems in knowing when to compute. Business and consumer related problems such as computing taxes and investment returns, determining interest on installment buying, and planning budgets proved to be difficult for many students. Estimating results of a computation was an especially difficult task. Table 1 contains a description of a sample of test items used in the Giesbrecht study and the corresponding results for the grade 12 students. The item wording and numbers have been modified to comply with copyright regulations.

Although it is gratifying to know that, according to the results of this study, the majority of senior high school students appear to be reasonably numerate, there are likely large numbers of Saskatchewan students who are functionally innumerate because they did not complete their schooling. A study of early school leavers in Saskatchewan

Table 1: Selected Test Items and Results from Giesbrecht's Study Involving 765 Saskatchewan Grade 12 Students

Description of Test Item[1]	% correct
$250 deposited at 4%. Calculate total after 2 years assuming no further deposits or withdrawals.	37
Given list of annual budget costs (Food $1250, Clothing, $300, etc.), determine the weekly job earnings needed to meet this budget, assume 50 working weeks.	74
Find 3% of 400.	83
20% of a number is 40, find the number.	72
Loan of $100 for one year. Service charge of $1 per month. Calculate yearly interest as percent.	64
200 divided by 0.204 is approximately ?	67

[1] Each test question used a multiple choice format.

(Cipywnyk, Pawlovich, & Randhawa, 1983) estimated that 31% of grade 8 students leave school before completing their high school education (p. 63). The highest early leaving rate was approximately 39% for boys from rural schools.

The great majority of these early school leavers performed poorly at school. For example, 75% of the early school leavers in 1980-81 were performing academically at a below average level (an average mark of D or F) in the year prior to leaving school. Consequently, an alarmingly high number of Saskatchewan young people attempt to enter the workforce without completing their schooling and are likely to be considered insufficiently numerate for many of the jobs they seek and the careers they intend to pursue.

More ominous are the forecasts that the already high unemployment figure of 14% for Canadian youth will continue to increase in subsequent years. Unemployment has reached a stage where many young people have never had regular employment. The numeracy skills of these chronically unemployed youth can be expected to diminish further if there are no opportunities to use and learn mathematics on the job. Never having regular work and denied appropriate upgrading opportunities, the unemployed become virtually unemployable.

Although opinion surveys and achievement studies provide useful information, they skirt this crucial question: what numeracy skills are required for today's jobs and careers? It is clearly inadequate to collect information about the numeracy levels of students without knowing the demands facing these students when they leave school. As Saskatchewan begins to re-examine and revise its school offerings, it is more important than ever to have information about the requirements of employment to guide decisions about the addition and deletion of topics and their attendant understanding and skills.

Long and Herr (1973) suggest that, although most teachers of mathematics are aware of the mathematical competencies needed for the highly qualified professions whose training programs are usually housed in the universities, it is:

> . . . less probable that teachers of mathematics can speak with equal confidence of the mathematical applications that are important to skilled and craft occupations or of the vocational education courses that are preparatory to entering such occupations.
>
> Finally, it is plausible that certain mathematical functions occur with more frequency or are of more priority in various occupational specialities than are other functions. Documented knowledge of such information could be used both as a stimulant to

student motivation and to teacher planning. (p. 61)

Fortunately, in recent years, a few attempts have been made to discover in an objective way what mathematics is presently needed in different areas of employment.

Surveys of Numeracy Needs for Employment

Many studies have attempted to identify needed numeracy skills for jobs by surveying employers, employees, supervisors, personnel officers, and vocational teachers. Often the information provided by some surveys is too general to be useful. For example, one study (Morris & Cope, 1982) reported that Australian employers were looking for workers who had the following abilities or knowledge: "able to work with figures," "a bit of maths," "numbers," and "mathematics to a basic degree" (p. 17).

Other surveys usually elicit more specificity from the employers. In 1976, McIntosh asked 700 employers to indicate the importance of various mathematical topics listed on a questionnaire. The majority of the 100 employers who returned the questionnaire expected their employees to be able to compute with whole numbers and decimals, add and subtract money amounts, add and subtract fractions, calculate mentally, use tables, tell time, measure length and weight, and use SI units.

Richards (1978) interviewed employers from a variety of industries in London. The basic numeracy requirements for these jobs were computing with whole numbers, understanding fractions and decimals, simple knowledge of ratios, proportions and percentages, the ability to read graphs and tables, and facility with estimation and approximation. He also noted that the ability to use an electronic calculator was commonly reported as a desirable job skill.

In 1978, Swinson surveyed Australian employers in Queensland regarding the mathematics needed by students leaving school. He asked employers to rank a list of numeracy skills in order of importance for workers in their field. The majority of employers who returned the questionnaire - only a dismal 145 out of 2500 - identified the following skills as being "very useful": listed in order of importance, they are simple addition, subtraction, multiplication and division of whole numbers; mental arithmetic; calculating with decimals; using a calculator; working with percentages.

Closer to home, Smith (1978) attempted to identify the generic skills required by Canadian workers in various jobs. Over a period of several years, he surveyed workers in 49 occupations and supervisors in 28 occupations. The definition of generic skills and the purposes of the study were provided in the report:

> Generic skills are those job behaviours which are actively used in work performance, which are transferable from one job or occupation to another and which are needed for promotion to supervisory status. By this definition most communication, simple arithmetic and reasoning skills are highly generic, while advanced algebra and science skills are not.
>
> Simply stated, the generic approach is an attempt to examine the communality [sic] of skills rather than the uniqueness of occupations. It is an attempt to determine how much electricians, carpenters and secretaries are alike in skill usage. It is also an attempt to discover which skills, commonly taught, are rarely used in job performance. (p. 3)

Smith found that computation and measurement skills are common everyday requirements for most workers. Most importantly, he found that mathematics at work is always measurement related: in other words, computations such as adding 1/16, 1/4, and 1/2 are never done without considering units of measurement. Workers are also expected to be accurate and verify their calculations and measurements. Charts and tables are used frequently by most workers. Increasingly, employers are providing their employees with calculators to promote productivity. Drawings and sketches are frequently used. Although graph reading and interpreting are much in evidence, few workers ever construct graphical displays.

The specialized technologies that Smith surveyed required a knowledge of elementary algebraic and trigonometric notions, but rarely is such knowledge needed by most other workers. The use of algebra usually involves formula substitution (e.g. calculating a voltage drop using the formula $E = I \times R$) and, in most cases, formulas are provided.

"Ready reckoners" are often used to eliminate esoteric calculations: looking up mortgage payments, sales tax, material strengths, etc. As his study was completed before Canada changed to the SI (metric) system, this type of knowledge was ignored. Smith predicted, however, a corresponding decline in the use of fractions as the SI system is adopted. Tables 2 and 3 contain a summary of his findings.

Recently Morris and Cope (1982) completed a massively detailed study of the literacy and numeracy demands on school leavers entering selected apprenticeships in Queensland, Australia. For each of the following trade areas, apprentices, tradespersons, supervisors or foremen, and employment officers were interviewed: electrical fitting and mechanics, fitters and machinists, motor mechanics, carpenters and joiners, painters, cooks, hairdressers, boilermakers, plumbers and diesel fitters. Interviews of subjects working in a trade were continued until the interviewers thought that no significant new information was being added in successive interviews. The results, which are very similar to Smith's, have been summarized in Table 4, on the following page.

One noteworthy finding of the Morris and Cope study is the frequent

Table 2: Generic Mathematics and Reasoning Skills Identified in Smith's Study of Canadian Occupations

SKILL NEED - Nil * - Few ** - Many *** - Almost all	OCCUPATIONAL FAMILIES										
	Clerical	Engineering Technologists	Medical/ Health	Sales	Service	Machining	Fabricating, Assembly and Repair	Construction	Motor Transport	Farmers	Supervisors
ARITHMETIC											
Whole Numbers	***	***	***	***	***	***	***	***	***	***	***
Fractions	***	***	***	***	**	***	***	***	**	***	***
Decimals	***	***	**	**	**	**	***	**	**	***	***
Percent	***	***	**	***	**	**	***	**	**	***	***
Ratio/Proportion	*	***	**	**	**	**	***	**	*	***	***
Mixed Operations	***	***	**	***	**	***	***	**	**	***	***
Measure: Time	***	***	***	*	***	***	***	***	***	***	***
Weight	**	***	***	**	**	*	**	**	***	***	***
Distance	*	***	***	**	**	***	***	***	***	***	***
Capacity	*	***	***	*	**	*	**	***	**	***	***
GEOMETRY											
Calculate: Perimeter		***	*	***	**	***	**	**	***	***	***
Area		***	*	***	**	***	**	***	**	***	***
Volume	*	***	*	**	**	***	**	**	***	***	***
Draw/Sketch		***	*	*	**	***	***	**	**	***	***
Read: Scale Drawings		***		**	*	***	***	**	**	***	***
Assembly Drawings	*	***		*	*	***	***	**	**	***	***
Schematic Drawings		***		*	*	**	***	**	**	***	***
Graphs	*	***	**	**	*	*	*	*			***
Draw: Graphs		***	**	*				*			***
Scale Drawings		***			*	**	*	**			***
ESTIMATING											
Time	***	***	***	**	***	***	***	***	***	***	***
Weight	**	***	**	**	**	***	**	***	***	***	***
Distance	**	***	***	**	**	***	***	***	***	***	***
Area	*	***	*	**	**	***	***	***	***	***	***
Capacity	*	**	***		**	**	**	**	***	***	***
Cubic Measure	*	***	***		**	*	**	**	**	***	***
Costs	**	***	*	**	**	*	**	**	***	***	***
INTERMEDIATE MATHEMATICS											
Solve Given Formulae	*	***	**	*	*	***	***	***		***	***
1 Variable Algebra	*	***	*	*		*	*	*			*
Algebra Powers/Roots		***		*			*	*			*
2 Variable Algebra		***									*
Quadratics		**									*
Logarithms		***	*		*	*					*
Trigonometry		***			*	*					*

Table 3: Core and Non-core Numeracy Clusters Identified in Smith's Study of Canadian Occupations

Core Clusters	Non-core Clusters
1. Read, write and count whole numbers	1. Compute ratios
2. Add and subtract whole numbers	2. Compute proportions
3. Multiply and divide whole numbers	3. Compute rate
4. Solve word problems with whole numbers	4. Compute principal
5. Round off whole numbers	5. Measure weight
6. Read and write fractions	6. Measure distance
7. Add and subtract fractions	7. Measure capacity
8. Multiply and divide fractions	8. Know geometric forms and figures
9. Solve word problems with fractions	9. Computation on angles
10. Compute dollars and cents	10. Draw/sketch geometric forms and figures
11. Read, write, and round off decimals	11. Compute perimeters
12. Multiply and divide decimals	12. Compute areas
13. Add and subtract decimals	13. Compute volumes
14. Solve word problems with	14. Read graphs
15. Read and write percents	15. Read scale drawings
16. Compute percentages	16. Read assembly drawings
17. Determine equivalents	17. Read schematic drawings
18. Know order of operations	18. Draw graphs
19. Solve word problems (mixed operations)	19. Measure from scale drawings
20. Do quick calculations	20. Draw to scale
21. Compute averages	21. Solve algebraic formulas
22. Read graduated scales	
23. Perform operations with time	
24. Operate calculator	

Table 4: Numeracy Demands on Graduating Students Entering Selected Apprenticeships in Queensland, Australia

	Electrical Fitter Mechanic	Fitter Machinist	Motor Mechanic	Carpenter and Joiner	Painters	Cooks	Hair-Dressers	Boiler Maker[1]	Plumber[1]	Diesel Fitter[1]
Mental Arithmetic	### **	### **	### **	### **	### **	### **	### **	### **	### **	### **
Four Arithmetical Operations	### **	### **	# *	# *	## **	## **		### **	### **	## **
Metrification	## **	### **	# *	# *		# **	### **	# **	## *	### **
Length	### **	### **	### **	### ***	### **		Estimation	Accurate # **	Accurate ### **	Accurate ### **
Area	# **	# **		### **	### **		Estimation	Accurate ### **	Estimate ## *	Estimate # *
Volume	# **	# *	# *	## **		Estimation Vital	Estimation	Estimate # *	Estimate # *	Estimate # *
Weight	## *	## **	Estimation	### **		### **	Estimation	Estimate # *	Accurate # **	Estimate ### **
Angles	# **	### **	### **	### **			Estimation	### **	### **	Estimate # *
Fractions	### **	### **	### **	### **		Estimation	### **	### **	### **	## **
Decimals	### **	### **	### **	### **		# *		### **	### **	### **
Conversion Fraction/Decimal	# *	### ***					### *	# *		# *
Use of Calculator	## ***	### **	### **	# *		## **		# *		# *
Use of Slide Ruler							# *			
Percentages	# ***	# *	### **		Concept Useful	## ***	Estimation		# *	## **
Ratio	## **	# *	# **	### **	Concept Useful	Estimation	# *		# *	## **
Interpretation Graphs	# **	# **		# *			## *			## **
Interpretation Tables/Figures	## **	### **	### **	## **		### **	# **	### **	## **	### **
Statistics										
Technical Drawing Interpretation	### **	### **	# *	### **	## **			### **	### **	## **
Estimation Error Length/Area	### *	### **		### **	### **		### **	o	o	o
Estimation Error Figures/Money	# **		# *		# *	### **	### *	o	o	o
Tolerances	### **	### **	### **	### **				### **	## **	### **
Simple Geometry	# **	### **	### **	### **			Estimation	### **	# *	# *
Simple Trigonometry	## **	### **						# *		
Simple Algebra	# **	# *								
Speeds	## **	### **	### **					# *	# *	### **
Transformation Formula	# **	# *								

USAGE		IMPORTANCE	
###	Daily	**	Vital
##	Weekly	*	Supplementary
#	Occasionally		

NOTE: 1. When these trades were surveyed the questionnaire had been changed, omitting questions found to be ambiguous in the first trial - questions omitted are indicated by an 'o'.

need in most trades to convert from SI (metric) to the Imperial system. Older or American tools and products necessitate these conversions. Conversion charts are used to accomplish these unit changes from one system to another, although the ability to estimate measures in both systems and to perform quick conversions are to a tradeperson's advantage. The ability to handle straightforward mental calculations is considered vital in most areas.

There are several shortcomings in many of these studies of numeracy demands. The low rate of return of questionnaires used in some studies leaves many vital questions unanswered. Are the responses of the employers who returned the questionnaires representative of the ones who did not? Did some employers not bother to complete a questionnaire because their workers had little need for numeracy skills (e.g. routine, production line work)? Surveying only employers' opinions also weakens the findings of a study. Employers in large companies are usually represented by a personnel officer who, in some cases, may be quite unfamiliar with the daily routines of workers. Even if they were once familiar with a trade, they may have forgotten aspects or not kept up with new ideas or techniques.

The most serious flaw in these surveys is that employers often equate the numeracy requirements of the screening tests they use or the training programs they or others provide with those skills needed for successful job performance. Hiscox (1980) found that many topics taught at vocational or technical schools are often unneeded by workers. She also found that many large firms administer arithmetic tests to prospective employees despite a lack of evidence that the arithmetic skills being tested are needed for successful job performance. In many cases, the tests are being used to screen out applicants.

Studies of On-the-Job Numeracy

The most effective way to determine the numeracy demands of various jobs is to observe workers performing their daily routines. For obvious reasons, such studies are very difficult to conduct and few are done. Fortunately, a few studies can be found. The original reports prepared by Hiscox (1980) and Pirie (1981) and a summary of a study conducted by the University of Bath (Cockcroft, 1983) were examined and provided useful information. (For sake of brevity, the summary report of the University of Bath study will be referred to as the Bath study.) After a brief discussion of the methodology and sampling of each study, some general observations about numeracy on the job will be presented, followed by a discussion of the types of mathematics that workers commonly use.

Hiscox (1980) interviewed both employers and employees and observed on-the-job performances in a variety of industries within the Nottingham/Derby area: construction, manufacturing, agriculture, printing, catering, the retail and distributive trades, and health and hospital work. She deliberately ignored the engineering industry because much earlier work had been done in this sector. She also attempted to identify the type of training expected for each job and the selection tests used by the employers.

Pirie (1981) focussed on mathematics in medicine, particularly the on-the-job numeracy demands of English nurses serving or training in a variety of health-related fields. The data for the report was collected in a variety of ways: visits to hospitals and on-ward observations; interviews with nurses in training, qualified practitioners and tutors; analysis of entry requirements and tests; scrutiny of examination papers;

studies of textbooks; and an analysis of over 550 questionnaires administered in hospitals throughout England.

More than 90 companies and other establishments were visted in the Bath study. Observations and discussions with employees and discussions with the management and training staff were used to collect the data.

General Observation

The Bath study points out that those with a mathematical eye can describe many aspects of a job in mathematical terms. In manufacturing, for instance, stacking and packing can be described using geometrical ideas such as symmetry and translation. However, even if these ideas have been encountered in school, employees do not consciously use such notions to analyze the tasks they perform; most importantly, even if such tasks are viewed as mathematical instances, employees will not necessarily be able to do their jobs any better.

These studies emphasise that possessing knowledge of mathematical procedures and skills does not imply that workers know when and how to use them to help perform their jobs. Young workers, in particular, often need explicit instruction and practice in applying their school-learned knowledge to the tasks encountered on the job. Considerable patience is needed by employers and co-workers because young workers who would normally be able to accomplish a task sometimes cannot perform it under the pressures exerted by a new job.

Job procedures derived mathematically may not necessarily be the best way to perform a job. Workers can and do learn "rule-of-thumb" methods that are often more efficient than school-taught methods. For example, Hiscox points out that plumbing textbooks on the installation of eavestroughing give complicated formulas for the calculation of the bore and number of pipes needed for a roof of a given size and pitch, sometimes taking the local rainfall characteristics into account. Few experienced plumbers use these notions, however. The necessary sizes and numbers of pipes and eavestroughing are decided from experience and, as is common in most trades, "erring on the generous side when in doubt."

Such a finding illustrates an important difference between the way in which mathematics is used in employment and the way in which the same mathematics is often encountered in the classroom. Differences in use may be among the factors that have contributed to the employers' criticisms about school math voiced in recent years.

Hiscox found that most employers thought that young people entering the workforce are less numerate than they had been in the past. This decline is very much blamed on a perceived change in the subject matter of school mathematics programs. Employers have the mistaken belief that most of the more traditional topics such as basic arithmetic, percentages and fractions are no longer dealt with in any depth at school. Hiscox thinks that young workers who encounter difficulties in carrying out some mathematical process tend to excuse themselves by saying that they were never taught the process. She commented: "Whether or not this is genuine forgetfulness on the part of the school-leavers, the employers seem ready to believe them" (p. 66).

Most employers in all studies supplied one or two examples of employees whose insufficient numeracy skills affected their job performance and jeopardized their careers. One department store participating in the Hiscox study provided several examples. One girl who was ideally suited for a job in the music department was unable to work out the rebate when a customer exchanged a book costing 36p for a more expensive one costing 50p. One young person was dismissed because of an inability to reprice products for sale at half the original price: in other words, dividing a number by 2. Only three out of seventeen applicants were able to work out 50% of 8 pounds and the change from 1 pound for a 42 1/2p purchase (p. 142). This particular store adopted a policy of employing older women where possible because "they have to be paid more but are confident with this basic arithmetic" (p. 142). Hiscox commented:

> Few employers took the dismissal of a hardworking employee lightly and felt angry that weakness in mathematics should sometimes force that decision upon them. This was a further cause of discontent about the standard of school mathematics teaching. (pp. 87-88)

The comments of young employees showed that very often, but not always, those who had difficulty with routine mathematics did not like the subject of mathematics. Equally, success more often than not seemed to lead to a favourable attitude: "You enjoy it when you can do it," as one employee in the Bath study said.

Interestingly, the belief that mathematics was useful was shared by many of those who disliked mathematics as well as by those who liked it. Most of those interviewed in the Bath study, whether young or old, "saw mathematics in vocational utilitarian terms. Very few saw it as serving any wider purpose, especially of what might be termed a cultural or general educational kind" (Cockroft, 1983, p. 61).

Common On-the-Job Numeracy Demands

Job demands are highly idiosyncratic. The on-the-job numeracy demands of engineers, electronic technicians, retail workers, boilermakers, hairdressers and painters vary considerably. Despite this great variation, it is possible to describe in general terms the types and levels of mathematics needed by the majority of workers and how they are carried out.

Whole number arithmetic: Adding, subtracting, multiplying and dividing with whole numbers are done frequently in all jobs, but rarely are large numbers involved. Calculations involving multiplying and dividing frequently involve one-digit factors and divisors: e.g.

calculate the number of items in 5 boxes each containing 24. Rarely does a worker use pencil-and-paper methods to divide and multiply by multidigit numbers. Multiplying and dividing larger numbers seems to be common only in occupations where similar calculations with decimal fractions are met.

Both the Bath and Hiscox studies reported that workers frequently ignore the customary methods of calculation taught in school. Instead, employees use a variety of idiosyncratic methods of calculation, especially for long division and multiplication. These methods are either self-invented or have been passed on by a fellow worker. There appears to be a preference for carrying out a calculation in a succession of relatively short stages rather than a more efficient and perhaps quicker method done without understanding.

The Bath study provides the following example of an unconventional algorithm. A boy who left school early had the responsibility of performing complex checks on whether he was up to quota. He used the following steps to calculate 7 x 96:

$3 \times 96 = 288$

$$\begin{array}{rr} & 288 \\ + & 288 \\ \hline & 576 \\ + & 96 \\ \hline & 672 \end{array}$$

Although this employee was unfamiliar with the seven times table, he clearly understood the notion of "7 times" a number. Cockcroft (1983) comments: "This is one of the many examples we found where people resorted to longer methods with which they evidently felt confident, rather than using what a mathematics teacher might have thought to be the obvious direct method" (p. 20).

Calculators. Performing a calculation is one of the most frequently called for tasks on the job. To increase speed and accuracy and ultimately productivity, most employers encourage their workers to use calculators. There were a few noteworthy exceptions noted in the studies of on-the-job numeracy.

Some employers who refuse to allow their employees to use calculators seem to believe that those who use calculators suffer from a character deficiency. These employers attach great importance to the methods they were taught to use and ensuring that their employees use these methods has become almost a mission.

New apprentices in some technical areas reported that they were issued log tables instead of the scientific calculators they had been using in training college. Hiscox noted this particularly unrealistic view of calculator use by one bank manager:

> The opposition to calculators seems to be somewhat prejudiced. The manager made several statements to the effect of: "I had to do much more tedious things than this" and seemed to think that this arithmetic was somehow character building - a necessary attainment for any decent citizen (p. 149).

She noted that this bank was currently running advertisements for staff suggesting that little mathematics was now required.

Price reported that in one large hospital she visited, the head nurses and hospital managers were almost unanimously opposed to allowing nurses to use calculators. The reasons were not based on the possibility of producing undetected erroneous answers that might be harmful to patients. Instead, their opposition was based on emotive thinking: "Arithmetic is good for them"; "I always do my addition on a piece of paper"; and "Patients might lose confidence in the nurses."

Other employers object to the calculator on the grounds of the image created for the customer. Hiscox reported that one of the more image conscious department stores in her study frowned on the use of calculators by the staff, except when demonstrating them for sale! A similar concern for image was encountered in banks.

There were some legitimate concerns raised by some employers, however. Because the majority of workers who use calculators regularly have not been trained in their use either at school or on the job, calculators are frequently not being used in the most effective manner.

In one factory where pocket calculators were widely used, the major criticism voiced by the personnel manager was that employees often fail to notice errors in calculation. According to him, his workers had no idea of how to approximate to check the answer displayed on the calculator: for example, an incorrectly placed decimal point often went unnoticed, resulting in an improbably long or short length of material (Hiscox, p. 124).

The number of decimal places displayed by a calculator gives some employees difficulties: e.g. determining a price calculation when the calculator displays a number like 851.58971. Using a calculator to convert a fraction to a decimal (using the division key) and interpreting the displayed answer (e.g. does 8.3333333 mean 8 or 9 eggs should be used in a recipe?) troubled some employees.

Mental arithmetic. Calculating mentally is a surprisingly frequent work task, despite the proliferation of calculators. In almost all jobs, the ability to carry out straightforward calculations in the head is highly valued and the inability to do this is a frequent cause for complaint by both employers and employees. Interestingly, many young workers complain that their ability to calculate mentally that they had possessed in their early school years had been allowed to atrophy as a result of lack of practice in their secondary school years.

Estimation. The ability to estimate is a highly valued work skill. Estimation on the job seems to appear in several forms. Numerical estimation is used to detect calculation errors whether produced by pencil-and-paper methods or a calculator. For example, an answer of $1387.50 is suspiciously high for a cost calculation of 75 items at $1.85 each because 75 @ $2.00 costs only $150. Employers complain that beginning workers often lack a "number sense" and rarely use numerical estimation to detect errors and, consequently, their accuracy suffers.

Experienced workers also use current knowledge gained from the job to detect errors. For example, Pirie reported that experienced nurses know immediately that a calculated dosage of 150 ml is excessively large because most adult drugs are dispensed in 5 ml quantities. This is a reason why some young workers such as beginning nurses, who obviously cannot depend on current knowledge to gauge the reasonableness of produced calculation, need to be supervised carefully. Using knowledge acquired on the job to estimate is another example of how real-world experiences can help promote numeracy.

Another form of estimation involves measurement. Experienced workers often have to make decisions based on visual inspection rather than on accurate measuring, particularly in situations where measurement is difficult or awkward, where trial and error is possible, or where tolerances are large. In these situations, a "feel" for a measurement unit is essential: "That board is about 5 m in length"; "I need either a 7/16 or 1/2 inch wrench"; "You'll need about 32 litres of paint." Visualisation and visual interpretation are also needed: "That container will hold more than the other one." Although skills of measurement estimation develop on the job, employers frequently complain that young workers do not have a good "feel" for measurement, whether SI or imperial units are involved.

Measurement. A knowledge of measurement is a vital factor in the successful performance in almost all jobs, even if measuring instruments are rarely used. In some jobs, the worker needs to be actively involved in the measurement process: e.g. measuring lengths, masses, weights, volumes, temperatures, etc. In these cases, a knowledge of what measuring instrument to use and how to use it is absolutely necessary. The worker must also be aware of how accurate a measurement must be. In small construction jobs, accuracy might be measured to the nearest bucketful of cement; in machining, accuracy might be measured to the nearest ten thousandth of an inch.

Some employees, mainly white collar workers, need to be keenly aware of the meaning of measurement, although they seldom take an active part in the measurement process. These workers can be considered to be consumers rather than producers of measurement information. Ordering and costing, calculating profit margins, predicting voter trends, monitoring seasonly adjusted unemployment rates, analysing divorce rates, and evaluating the success of a program are a few of the many tasks requiring an interpretation of carefully gathered measurement information.

Most countries like Canada have changed from the Imperial to the SI measurement system and most industries expect entering workers to be familiar with the common SI units. Unfortunately, there appears to be a need for some employees to have a working knowledge of the common Imperial units in addition to a knowledge of the SI system.

Many firms using older machinery are still using Imperial measurements. There is also a need to convert from one measurement system to another: e.g. translating a customer's (non-metric) requirements into the metric equivalents needed by the company. Hiscox found that, where a conversion between systems was needed, it was accomplished by referring to a conversion chart.

Fractions and percentages. Calculations involving decimal fractions to two places are common in relation to money, especially addition and subtraction. The general use of decimal fractions has been increased by the use of SI units. Except for engineering, rarely is multiplication and division by decimal fractions used. Hiscox found no example of somebody using long multiplication or division, apart from the training establishments.

Recognizing fractional parts such as halves, quarters, etc. is needed by most people, but since the changeover to SI few common fractions are seen in calculations. Only simple conversions from fractions to decimals and decimals to fractions were seen: e.g. 0.25 = 1/4, 0.5 = 1/2. Apart from these examples, no situations requiring conversion were found. Engineers, however, make heavy use of conversions.

Common fractions when used on the job almost always have denominators that are powers of 2: namely, 2, 4, 8, 16, . . ., 64. Operating on such fractions as 4/7 + 3/5 is rarely needed and, in the very rare instances fractions need to be multiplied or divided, it is usual to use a calculator to convert to a decimal before performing the operation. Hiscox identified the need to ensure that school-leavers are able to enter fractions into a calculator: e.g. 3 divided by 4 is entered as 3/4. Recognizing that 0.55 is approximately 1/2 will become increasingly important if calculators continue to be used.

Most people who work with fractions use the visibly present marks on the rule to add and subtract fractions and to determine equivalent fractions. When calculations are carried out using pencil and paper, common denominators never have to be worked out because they are readily apparent from the denominators used. For example, in adding 3 1/4 and 5 3/16, the denominator 16 will be used.

The Bath study also reported that craftsmen sometimes use self-developed methods as an alternative to the methods taught in school. This example taken from Cockcroft (1983) illustrates how one craftsman added 3/16 and 5/64:

$$\begin{aligned} 3/16 + 5/64 &= 3/16 + 1/16 + 1/64 \\ &= 4/16 + 1/64 \\ &= 16/64 + 1/64 = 17/64 \end{aligned}$$

As the Bath study reported: "It [the craftsman's method] could be described as very mathematical but it is not necessarily the method which would be reinforced in schools" (Cockcroft, 1983, p. 22).

The use of percentages is widespread in laboratories and offices. Most uses involve money (e.g. calculating taxes, profit and loss, commissions and discount, interest rates, etc.) and those who do use percentages need to very familiar with the various ways of handling them. Employers complain frequently that many workers, including managers and supervisors, encounter difficulties with percentage problems: e.g.

realizing that an increase or decrease of 15% can be calculated by multiplying by 1.15 or .85.

Other topics. Reading tables, charts, graphs, scales and dials is required for almost all jobs. Few employees need to construct graphs and charts, however. As computers continue to enter the workplace, the interpretation of tables of figures and visually displayed graphs and charts will become commonplace. The introduction of digital displays will continue to reduce the need to read analog scales, but there will be a greater need to appreciate number relationships (e.g. 1485 rpm is approximately 1500 rpm).

Rarely do most workers make use of algebra, geometry and trigonometry and those who do are employed in technical fields requiring a relatively sophisticated knowledge of mathematics. When algebra is encountered at work, it usually involves no more than substituting numbers for variables expressed by letters or words: e.g. child's dose = Age x Adult dose/Age + 12. Very few workers use algebraic ideas to transform one formula into another related formula. Instead, a needed formula is usually looked up in a reference book. There is little call for calculating powers or roots and when such calculations are needed a calculator is used.

A knowledge of the elementary properties of the common geometric figures and the associated terminology is adequate for most jobs. The construction industry makes an occasional use of Pythagoras' theorem, usually to square up walls and partitions. Only the engineering industry makes relatively frequent use of trigonometry.

IV. NUMERACY DEMANDS IN THE FUTURE

A shortcoming of all studies of everyday and on-the-job numeracy reviewed in this report is that only a snapshot of the present is provided. In a static society, the findings provided by these studies might remain valid for years. However, we live in a rapidly changing and competitive world where many skills needed to function in the past have become or soon will become outmoded and unnecessary. Recent trends and developments in consumer and work technology suggest that, for most people, the numeracy demands of adult life will likely be minimal. There are a number of ways that technology can affect and is currently affecting the numeracy requirements needed in adult life.

The most obvious way to eliminate the need for numerate workers is to use technology to replace all or some of the skills traditionally done by workers. Besides routine production line work, which never did require numerate workers, technology is appearing in other fields that, in the past, required very numerate and skilled workers. For example, machinists, lathe operators and draftsmen have traditionally required a considerable knowledge of arithmetic to perform their jobs. Machinists and lathe operators frequently need to perform the following mathematically related tasks: computing sizes, reading vernier scales and machine settings, reading technical drawings, calculating rotational speeds, measuring stock, and calculating tolerances. Draftsmen have always needed to be very numerate, requiring a solid knowledge of projections, scales, ratios and trigonometry to be successful.

Computer-aided manufacturing (CAM) and computer-aided designing (CAD) lessens the need for these types of numerate workers. Robots and computers are now being used by many firms to control lathes previously operated by well-trained and experienced human workers. Similarly, computer-aided design has reduced and will continue to reduce the need for skilled draftsmen. In 1982, the Victoria Times-Colonist interviewed one Victoria businessmen whose firms provided architectural drawings for the construction industry. The equipment his firm was using at the time of the newspaper article made architectural drawings in about one third the time it took using conventional pen-and-ink work. He cited the case of an 18-year-old girl who, without any previous drafting or video-terminal experience, learned skills comparable to a draftsman trained for 2 years. Her period of training took one month and she became useful to the firm after 2 months. A special feature of some CAD systems is the ability to interface with various manufacturing and modeling systems. With RolandCorp's Computer-Aided Modeling Machine (CAMM-3), for example, a prototype of a design can be readily produced by routing the design data from the CAD software to the modeling machine. Once a design has been constructed on a videoscreen and verified by producing a design on the plotter, a plastic, metal, wax, or wood model can be produced simply by relaying the plotter information to the CAMM-3 software where it is converted into instructions for cutting, drilling, and milling. Companies in Saskatchewan are now beginning to use CAD/CAM technology.

In the retail trade, cashiers at one time had to read or remember prices, enter them into a cash register, count money and make change. Now the use of optical readers, electronic cash registers and automatic change dispensers has significantly reduced the numeracy demand of this job. The automatic bank teller is another form of technology that will have a significant effect on the skills of workers needed by banks.

Prefabrication, prepackaging and modularization is another form of technology that eliminates the need for many of the traditional skills used by workers. Parts of houses and even entire houses can be delivered to a site prefabicated, reducing the measurement-related tasks normally done by a carpenter. Some Japanese companies manufacture modular buildings where even the electrical, telephone, and cable television wiring are done in the factory. Instead of the usual variety of tradesmen that are presently needed to construct a conventional building, modular buildings can be assembled on site by a specially trained crew who need very few numeracy skills. In the health fields, pre-packed drugs lessen the need for pharmacists and nurses to calculate drug doses and solution strengths.

Specialized calculators, similar to the presently available scientific, engineering and financial calculators, and computer software programs could significantly reduce the need for workers to remember routine procedures and formulas. For example, nurses have to know and use such formulas as the following: fluid requirement = K x expected body weight/number of feeds. According to Pirie, the value of K varies and this further complicates the calculation. A calculator or software program specifically designed for the health fields would not only eliminate the need for remembering formulas, procedures and their associated details but, if prompts such as "enter the age in months" or "enter the mass in kilograms" were also provided, accuracy could be improved as well. As a further safeguard against errors, the calculator or computer could check the reasonableness of an answer by referring to current knowledge stored in its memory. In the case of a suspected error, a device could be programmed to provide a warning message: e.g. "Dosage too high for a child. Redo the procedure." Specialized calculators and programs could be useful in all fields where routine arithmetical procedures are found.

The trend to replace the routine and repetitive aspects of work with highly efficient forms of technology can be expected to continue, displacing workers who previously handled these routine tasks and degrading previously valued intellectual and manual skills. As the traditional jobs involving the production and distribution of goods disappear or become less labour-intensive, there is expected to be a corresponding increase in new occupations, many of which will involve the analysis and production of information. The new workers will be "mind-workers" - to borrow a term coined by Alvin Toffler - who need to possess sophisticated intellectual tools to function in the new world of work.

Many of these workers will have to be very well educated, particularly for fields that require an extensive technical knowledge. Of course, very few people have the interest, ambition or ability to acquire the specialized knowledge needed by this "technocracy" and the majority of people will not be able to enter the technical, professional and managerial fields that are developed.

What level of numeracy will be needed by our young people to function in this new society? Perhaps, the numeracy requirements of future ordinary life might be as minimal as de Bono (1979) suggests: namely, simple addition and subtraction; the use decimal notation; multiplication and simple division; the use of pocket calculator; the handling of elementary percentages; the mathematics of measurement at a basic level; and the use of computers (p. 213). Obviously, it is risky to predict the numeracy skills needed by the masses of young people who leave our schools to enter the everyday world of the future when no one knows for certain what jobs will be available. However, it can be said with certainty that the trend to degrade algorithmic knowledge will continue and requiring children to learn arithmetical skills that have or will soon become automated is wasteful, futile, and perhaps, hypocritical. The general understandings and specific competencies needed by a numerate person to handle the demands of future ordinary life are listed in Appendix 1. How numeracy can best be promoted in school is the subject of Part 5 of this report.

V. PROMOTING NUMERACY IN THE SCHOOLS

The primary purpose of this report is to identify the mathematical understandings and competencies essential for functioning effectively in quantitative situations that students will likely encounter when they graduate from school. In this final part, an attempt will be made to list those mathematical ideas needed by people in acting confidently and intelligently in everyday practical situations. Regardless of the future advances in technology, all members of our society should be sufficiently numerate to be able to do the following tasks independently and with a minimum of intellectual effort:

1. *Carry out the mathematical routines necessary for wise consuming*: Reading prices; calculating costs; handling money; determining the better buy; preparing simple budgets; completing forms; understanding credit plans; understanding product claims and specifications; reading bills; writing cheques.

2. *Carry out the mathematical routines necessary in homemaking*: Measuring; reading and following recipes; reading guages and instruments; planning renovations and repairs.

3. *Function in most non-technical work settings*: Handling instruments and reading their settings; reading and completing forms; measuring; working with money, percentages and measurement units; working with calculating devices; reading and understanding information organized in charts, graphs and tables; reading plans; reading and following directions.

4. *Read and interpret quantitative information found in newspapers, popular magazines, government, political and business publications and to evaluate arguments based on such information*: Working with graphs, charts

and tables; understanding common quantitative terminology; knowing simple statistical techniques and methods; understanding the fundamentals of surveys and polls; understanding simple expressions of probability and their various common uses.

5. Know sufficient mathematics to use and work with commonly seen technological devices: Driving a car; working with tools, calculators and simple appliances; running simple computer software programs.

6. Participate and enjoy leisure activities, games, and sports: Using scoring schemes; understanding simple statistics.

7. Adapt to new mathematical demands as needed: Learning new employment skills; using instruction and training manuals.

The mathematical understandings and competencies required by a numerate person to handle these real-world tasks are presented in Appendix 1. They have been derived largely from the findings of the studies of the mathematical demands of everyday life reviewed earlier in Parts 2, 3 and 4, although a variety of curriculum guides, instructional materials, scholarly articles, books and technical reports were also consulted.

Five major categories of mathematical understandings are identified: (1) Knowing how to use mathematics to solve problems; (2) Knowing how to perform calculations; (3) Knowing how to measure; (4) Knowing how to work with space and shape; (5) Knowing how to analyze and interpret quantitative data and arguments based on this information. General and specific competencies are listed under each of the major categories of understanding. No attempt has been made to sequence these competencies.

The list of understandings and competencies can be used in several ways by curriculum developers and teachers of most subject areas. Firstly, it can be used to determine those aspects of numeracy that can be incorporated into a particular subject area. Secondly, the list and the discussion to follow can be used to design appropriate learning activities to ensure that numeracy is promoted in almost all subject areas and to help students transfer their knowledge learned in one subject to that learned in another. Too frequently, students see little in common between mathematical ideas presented in the mathematics classroom and those seen in other subject areas: e.g. learning a formal method in a mathematics course for working with ratios and proportions, but not being able to apply this learning when working with ratios seen in social studies (map scales), in science (density, rates) and in industrial arts (converting from SI to customary units). Thirdly, the list can be used to evaluate the adequacy of teaching materials such as textbooks, workbooks and tests in promoting numeracy. It should be emphasised that many of the more important aspects of numeracy cannot be adequately tested by pencil-and-paper tests.

The contents of the list should not be regarded as definitive in every detail. As with any list purporting to identify essential life-skills, regular review of its adequacy will be necessary. Nor can a claim be made that persons must master all competencies before they can be considered numerate. As in distinguishing between literate and illiterate people, there is no clear demarcation between numeracy and innumeracy.

An attempt has been made to identify those competencies and understandings that are still and will likely continue to be important for students to acquire regardless of their capabilities and ultimate career goals. However, it must be emphasised that some students take a long time not only to become familiar with mathematical ideas but to develop confidence in using them. Those who found it relatively easy to develop the confidence and facility to deal with ideas about numbers should not underestimate the difficulties that some people experience and the anxieties they suffer. Nor should the extent of the help that can be required to help those having difficulties in understanding and using mathematics effectively be underestimated.

The understandings and competencies identified should not be misinterpreted as representing the entire content of the K-12 mathematics program. The list should not be seen as limiting the range of work that should be attempted by more capable students who are interested in learning advanced mathematical ideas and techniques contained in senior secondary mathematics and science courses. Although important, numeracy is only one goal of mathematics education.

Mathematics teachers will continue to be primarily responsible for developing an understanding of mathematical concepts, rules, principles, relationships and procedures, but numeracy must be supported in other subject areas. Almost all subjects can include some meaningful opportunities for students to apply what they know about mathematics. Considerable emphasis should be given to the solution of problems, the handling of commonly seen quanitative tasks, the analysis of mathematical information, and the evaluation of arguments based on this information.

Problem Solving

At the core of numeracy is the ability to carry out routine tasks and to approach non-routine problems with some confidence that a solution can be found. Consequently, schools cannot be considered to be doing a satisfactory job in promoting numeracy unless their graduating students know how and know when to apply the mathematics they have learned.

Currently schools spend too much time teaching children rules and algorithms whose connection to the solution of realistic problems is often not clearly understood. It is quite common, for example, to find children and adults who can multiply and divide with fractions and decimals but cannot recognize situations where such processes might be used appropriately.

In order to help children establish links between the symbols, language and rules of mathematics and their corresponding uses in the real world, it is essential that, whenever possible, mathematical abstractions be introduced and practised in a context of solving realistic problems. Otherwise, a separation of a mathematical procedure or process from its real-world roots can have the unintended effect of creating a disconnected view about mathematics (Hiebert, 1984).

The current overuse of workbook, textbook and teacher-constructed exercises that focus almost exclusively on symbol manipulation should be discouraged. Instead skills and routines should be practised as children participate in practical activities such as those illustrated in the following list placed roughly in order of increasing mathematical sophistication: (1) Learning to count by 1s by counting pennies or moving markers on a game board; (2) Learning to skip count by 5s and 10s by counting with nickels and dimes; (3) Learning to add numbers like 9 and 99 by adding a dime or dollar and then subtracting 1 cent; (4) Learning a quick rule for multiplying decimal numbers by a multiple of 10 by using a metre stick; (5) Learning to add and subtract decimal numbers using a metre stick or working with money; (6) Learning how to perform operations with simple fractions by considering how a measuring cup can be used to add recipe ingredients or how a ruler can be used to find the difference between two lengths; (7) Learning about decimal notation and relationships such as ordering by working with a variety of linear measuring tools including rulers, micrometers, feeler guages, vernier calipers, tapes & chains, rulers, and trundle wheels; (8) Learning the meaning and characteristics of averages by calculating sports statistics such as earned run, hitting, and bowling score averages; (9) Learning to work with ratios and proportions by studying gear relationships of a 10-speed bike, calculating currency exchanges, and converting recipe amounts; (10) Learning how devices and techniques used to measure the "squareness" of corners in buildings involve geometric ideas such as the Pythagorean relationship.

The ability to recognize an application of a mathematical idea is closely tied to conceptual development. More time in school must be spent in helping children develop intuitive ideas about mathematics well before the more formal aspects are introduced. For example, percentage problems can be better understood by considering percent as a fractional part of a dollar. Once such meaning has been established, practical problems such as finding the discount of 10% on a $25 dollar item can be greatly simplified and a number of methods of solution can be examined: by completing a table of values (10 ¢ discount on $1, 20 ¢ discount on $2, 30 ¢ discount on $3, ..., $2.50 discount on $25); by using reasoning such as 10 ¢ discount for every $1 and 25 x 10 ¢ = $2.50 discount, for every $25. These intuitive methods should be mastered before children are expected to use more formal mathematical procedures: in the case of discounts, by multiplying by a fraction or decimal equivalent of a percent.

In the elementary and middle grades, most children learn best if they are involved in meaningful activities requiring the use of manipulative materials. Older secondary students can also benefit from hands-on experiences with the instruments and tools seen around the home or in most job settings. A great deal of discussion and oral work as well as work with concrete materials should precede the introduction of abstractions expressed symbolically.

Saskatchewan Education can encourage the use of practical activities in all subject areas, particularly mathematics, by providing schools with a list of recommended manipulative materials and suggestions for their use. Most of the necessary practical equipment is neither elaborate nor expensive and is already available in many schools. Much of these materials can be gathered easily by teachers. However, manipulatives will be used only if they are in sufficient quantity and readily available. Teachers also need to be provided with sources of activities suggesting how these materials can be used to develop mathematical ideas.

Saskatchewan Education should consider arranging bulk purchases of useful commercial materials to reduce the costs to local boards. The costs of good equipment can be offset by a reduced need for commercially produced workbooks, expensive and often rarely used kits, and photocopied or duplicated worksheets. The following list of practical equipment is far from complete and is included for illustrative purposes only: (1) Commercial equipment - interlocking cubes, measuring cups, balance pans, tape measures, pegboards or geoboards, protractors, set squares, simple micrometers, base 10 blocks, compasses, money facsimiles, 3-D shapes, rulers, and calculators; (2) Teacher-gathered equipment - egg cartons, empty containers, objects for sorting & counting, grids, 100-charts, tongue depressors, money, scissors, paper plates and string.

Whenever students are given problems to solve, much of the focus of instruction should be on the understanding of the problem statement (What information is given? What information is needed? What information is superflous?), and what analytic tools or general problem-solving strategies might be profitably used (Should a diagram be drawn? Can the problem be acted out by using materials? Should a list or table of numbers be constructed? Should a guess be made and tested for its adequacy? Is there a simpler, but analogous problem that can be solved? Can the problem be considered as a sequence of simpler problems?). Students also have to be taught a number of ways to determine the reasonableness of a solution (Is the numerical value obviously too high or too low? Does the unit included in the answer make sense? Is the answer realistic?). Even after a correct solution has been determined and discussed, the analysis of the problem should continue. Teachers need to take the time to discuss other methods of solution, help students make any generalizations, and show how other problems can be created by changing some of the features of the original problem. A few problems studied in depth contribute far more to developing the ability to solve problems than many problems treated superficially.

If children are not given adequate time to think about a problem, are given little explicit instruction on useful analytic tools, are exposed to only the simplest of problems, or are forced to use formal procedures they don't understand, their approach to problem solving can become very mechanical and subject to the influence of the irrelevant surface features of a problem. For example, a 12-year-old girl who was given a multi-step word problem to solve, immediately responded, "I can't do that problem because there aren't two numbers. I can't get any questions without two numbers" (Hope & Baker, 1986, p. 7). No attempt was made to apply a helpful analytic tool like drawing a diagram. Another 11-year-old girl provided this explanation of her approach to solving problems, which is typical of many students: "Problem solving is easy. If there are more than two numbers, I always add; otherwise, I subtract. If I'm not sure if it's multiplication or division, I divide and, if there is a remainder, I multiply instead" (Hope & Baker, p. 7).

The types of problems that children are exposed to in school has a significant effect on how they learn to solve problems as well as on their attitude towards problem solving in general. In school, mathematical problem solving is often restricted to word problems where solutions involve a single operation or combination of operations. As a consequence of the overuse of word problems, children regard problems as simply "words disguising an operation" and they rarely take the time to read and understand a problem statement. Reinforced by their teacher's attitude towards problem solving, children believe that there is a "right" way to solve these problems: e.g. subtraction must be used even though adding could also provide a correct solution. When provided with a problem they find difficult or unusual, few children draw diagrams, make a table, or try to simplify a problem statement and, when an answer has been obtained, few will try another approach to confirm the accuracy or reasonableness of the initial answer.

Children cannot be blamed for a lack of thoughtful consideration of problems if they are presented continually with problems that are expected to be solved instantly by either adding, subtracting, multiplying or dividing the two numbers seen in a problem statement. The most efficient approach to solve problems seen in school is to choose an algorithm - usually the one most recently taught or practised - and search for some numbers to operate on.

Students also need to appreciate the differences between the mathematical problems posed in school and those actually seen in real life. In school life, the problems are carefully formulated to eliminate ambiguity and contain precisely the information necessary for solution. Usually only an exact solution is considered acceptable and the methods of solution incorporating the most sophisticated mathematical procedures are preferred to those that involve more intuitive reasoning.

In real life, problems are not always clearly specified and many decisions must be made before attempting to apply some mathematics. The problem solver usually has to pose or formulate the problem, determine what information needs to be obtained, decide on the "best" method for solving the problem, and determine the degree of required precision. Estimating the amount of carpeting for an irregularly-shaped room provides a good illustration of how school problems differ from real-life problems (Borasi, 1986).

Faced with the problem of determining the size of a carpet, the problem solver will have to acquire information about the room (its dimensions, its shape, etc.), perhaps, by measuring the dimensions with a tape measure and then drawing a rough sketch of the room. Also the way carpets are sold must be considered: What standard widths are available? What units of measurement are used to specify the width and area of carpeting? This type of practical information is rarely considered when school problems are presented.

The formulation of the problem must be continually refined: Is it necessary to calculate the area or can an alternative procedure be used? Is it important to determine the minimal amount of carpet needed or is a rough over-estimate sufficient? The solution to a problem will depend essentially upon the formulation chosen, and in any case it can be expected to be only approximate - an area calculation of 5.672 square metres, for example, would be considered an unrealistic answer because of its excessive precision and would be rounded to at least 6 square metres. Nor is there a standard method of solution for determining the size of the carpet: different people would likely use different methods. Some people will make a scale diagram of the room and cover it with scale models of carpeting lengths. Other people will calculate the area of individual sections of the room by applying their knowledge of area formulas. Almost all people deciding to calculate will use a calculator. A few people will resort to the simplest method for solving real problems; and simply seek outside help: in this case, arranging for a professional carpet layer to provide a free estimate.

The detection of errors in practical settings is a relatively complex process involving a person's knowledge about mathematical relationships and their knowledge gathered from many life experiences. An answer of 50.7 square metres of carpeting for a room, for example, will not be seen as unrealistically high unless a person has a feeling for the average size of a room in a house. Although this type of knowledge is usually acquired incidentally by most people as they carry out their daily lives, schools can help children considerably by ensuring they acquire a storehouse of concrete referents and a facility to use these referents to evaluate the reasonableness of a solution to a problem.

It is important that teachers include examples of real-life problem solving in all school subjects, especially those problems that require continual reformulation, involve the collecting of information, and allow students to try a number of methods of solution, including ones that may not appear to be mathematically elegant. Experiences with real-world problem solving can begin as early as grade 1 using simple examples such as considering ways to calculate change or deciding how many sheets of paper are needed by the class for a writing project. (For an example of a project designed to help middle grades' students use mathematics to solve everyday problems, see Lesh, 1985.)

If teachers are willing to search for instances of mathematics used outside school, the number of applications that can be found is considerable. Almanacs, books of records and newspapers provide an inexpensive source of voluminous and often interesting data that can serve as the basis for the study of a realistic problem. For example, a recent article in the Star-Phoenix sports page (July 4, 1987, p. D5) reported a study to determine if a livelier baseball was responsible for a substantial increase in home runs this year over the previous two seasons. Tests were done to determine the liveliness of 116 baseballs taken from every major league team. The tests consisted of dropping each baseball onto a steel plate. The apex of the bounce was marked by sighting against a scale. Each ball was dropped a minimum of 5 times. The high and low figures were eliminated and an average was taken of the other three drops. The average bounce in a 1977 test was 8.45 feet, while the average this year was 8.39 feet. The study concluded that there was essentially no difference between balls used now and those used in the recent past and, consequently, the construction of the ball was not a factor in explaining the increase in home runs.

The creative teacher could use this study to teach many aspects about problem solving including (1) defining what is meant by baseball liveliness; (2) developing a practical but valid test for measuring liveliness; (3) determining the sample of baseballs to be tested; (4) using the test results to lend support to or reject possible hypotheses; and (5) providing alternative explanations and hypotheses to explain an observation and identifying ways to test these consequent explanations and hypotheses. By involving students in a replication of the test, a great deal of practice in measuring and calculating would be provided and much would be learned about the nature of empirical analysis. Further learning would take place if this type of experimental procedure was also used to test the conflicting claims of manufacturers about the distance of their golf balls.

Popular magazines like Discovery, Popular Mechanics and Sports Illustrated usually contain some information for which mathematical analysis can be interesting. Specialized books such as those used by tradespeople, businesspersons, consumers, etc. provide many instances of applications that require little knowledge of the particular subject matter. Often these books suggest that "rules of thumb" developed by practitioners be used to carry out some procedures instead of the precisely stated rules seen in school: e.g. mixing cement, building stairs, bending pipe, measuring out recipe ingredients, etc. More attention should be paid in school to the informal and realistic methods of problem solving that often take place outside of school.

Two particularly good sources of mathematical applications are A Sourcebook of Applications of School Mathematics and Applications in School Mathematics available from the National Council of Teachers of Mathematics in Reston, Virginia. Both of these books contain examples of applications requiring a knowledge ranging from simple arithmetic to more advanced mathematics such as trigonometry and logarithms. The Challenge of the Unknown is a series of 6 videotapes or films that are designed to show elementary, junior high and secondary students how mathematics is used by people as a tool to make tasks easier. The videos are available from Karol Media while the accompanying teaching guide is published by W.W. Norton and Company.

No attempt should be made to use obviously contrived problems to demonstrate the relevancy of mathematics. An extreme example is the following problem that was one of many similar problems used by a teacher of second-year algebra in an apparent attempt to show students how standard algebraic techniques can be applied to real-world situations (Farrell & Farmer, 1980, p. 278):

> Sammy Nerd and his friend Ed Efrom are local pushers who have just received a new batch of heroin which is 92% pure heroin. Sammy and Ed, however, are interested in earning a little extra money and want to cut the stuff to 55% heroin. How much pure quinine must they add to the 50-ounce shipment to get the correct mixture?

Not only is the subject matter of the problem questionable, but problems like these are patently unrealistic: If a dilution procedure is undertaken by drug dealers, it is highly unlikely that formal algebraic techniques would be used or even known.

Although some researchers believe that the best advice for improving problem solving is to give students lots of problems to solve, practice in problem solving cannot be considered a substitute for carefully planned instruction. Students need to be provided with carefully guided instruction on how to analyze problems. The Mathematical Problem Solving Project (MPSP) outlined by Lester (1985) provides a useful model that all subject-matter teachers can use to guide their problem-solving instruction.

The instructional process is divided into three phases: (1) Problem

Presentation; (2) Solution Effort; (3) Problem Solution and Discussion. The behaviours of students and teacher and the intended learning outcomes are presented in Table 5 (Note: I have made a few modifications to the original table presented by Lester).

Lester reported that the final phase, Problem and Solution Discussion, was the least successful phase of the instructional model. Teachers were unsure of how directive they should be in modelling good strategies or in accepting student efforts no matter how naive (p. 49). Lester thought that this reaction was to be expected since it is difficult to teach students to analyze and generalize and rarely are teachers required by the mathematics textbooks they use to engage students in these types of higher-level thinking activities. Notwithstanding these difficulties, the MPSP did seem to improve the ability of students to analyse and solve problems and the instructional model used by the teachers in the study played a significant role in the students' increased ability to reason.

Calculation

Ever since the study of arithmetic became a compulsory school subject, learning to compute has been the major goal of instruction because it has been considered necessary for functioning in adult life. For many people, being numerate means being able to use paper and pencil and the standardized Hindu-Arabic place-holding algorithms to compute sums, differences, products and quotients.

Because of its perceived importance, teachers continue to spend a considerable amount of time trying to ensure that their students master pencil-and-paper computational methods: one study estimated that approximately 90% of the *instructional time in* arithmetic is spent in

Table 5: Desirable Teacher and Student Behaviours and Intended Outcomes for Three Phases of Problem-Solving Instruction

1. Problem Presentation

Students develop appropriate understanding of the problem.

Teacher Behaviour	Student Behaviour
Presents problem to class.	Reads and/or listens to problem statements.
Answers questions about problem comprehension.	Asks questions for clarification.
Asks questions to determine problem comprehension (What is given? What is needed? What is irrelevent?)	Records pertinent information.
Lists main points of problem.	
Organizes students to begin work on the problem.	

2. Solution Effort

Students develop and implement a plan for solving the problem.

Teacher Behaviour	Student Behaviour
Encourages students to share ideas with others.	Works alone or with peers on the problem.
Poses questions to refocus students' attention on relevant information.	Tries to employ various general problem-solving strategies that might prove useful.
Provides hints about useful strategies (drawing a diagram, making a table, etc.) when necessary.	Tests adequacy of solution by reading problem requirements again or by sharing the method with other students.
Encourages students to persevere.	

3. Problem Solution and Discussion

Students gain new "insights" into the problem and "good" problem-solving behaviour.

Teacher Behaviour	Student Behaviour
Allows a few students to demonstrate their solutions	Offers to discuss his/her solution.
Asks students to critique these solutions.	Listens/discusses other students' solutions and offers suggestions for improvement.
Ensures that students are aware of ways to check solutions (estimating, comparing with concrete referents, appealing to logic, etc.)	Attempts to identify generalizations.
Solicits alterative methods of solution (e.g. Could adding have been used as well?).	Attempts to analyze what worked and why.
Aids in bringing to light any possible generalizations.	Tries to provide examples of a related problem.
Asks students to identify how the original problem might be modified to create another problem.	Identifies incorrect reasoning, errors in procedure, or the use of incorrect information.

learning or practising computation.

But as the literature review of everyday arithmetic contained in this report indicates, placing heavy emphasis on the teaching of written computation is largely a waste of time and money, especially in the age of the inexpensive hand-held calculator. Adults rarely need to figure out complex calculations such as 1.87 x 2.45 and, when they do need to, a calculator is faster, more reliable and, in many instances, more accessible than computing with pencil and paper.

The pursuit of a high degree of computational proficiency could be justified if mastery required little time and effort. But, as the findings of recent provincial studies in British Columbia (Robitaille & O'Shea, 1985; Robitaille, 1981; Robitaille & Sherrill, 1977) and in Saskatchewan (Beamer, 1976; Giesbrecht, 1977; Randhawa, 1986) indicate, many students are unreliable calculators even after several years of instruction and practice. For example, Randhawa found that only about 20% of grade seven students could multiply and divide decimals accurately (p. 26). The grade ten sample showed little improvement in handling decimal and fraction arithmetic (p. 31). Many studies undertaken in several countries show that a significant number of students and adults never learn how to figure with decimals and fractions accurately. This lack of facility is not a recent phenomenon; the evidence of earlier studies indicates there has never been a "golden age of calculation."

To estimate the cost-effectiveness of the present focus on written computation, consider long division as an example. The time needed to achieve and maintain an adequate level of performance in long division has been estimated to take as long as two years of arithmetic instruction (Wheatley, 1980). Put another way, the cost of producing unreliable human "dividers" who rarely need to apply this skill outside of school is approximately $750 - $1000 per student.

A Saskatoon teacher provided me with the following illustration of how calculators can save instructional time and improve the accuracy of students. Grade 7 and 8 students were given a test involving pencil-and-paper computation of decimal numbers. After working with calculators for a few weeks, the test was re-administered, but this time calculators were provided. The results of the two test administrations are displayed in Table 6. Quite a startling improvement for a nominal investment of about $5.00 per student. Clearly, it is not cost effective anymore for schools to attempt to mass produce human calculators when inexpensive and more reliable electronic calculators are available.

It should also be noted that the time needed to teach children how to compute often has been gained at the expense of teaching them to know when to compute. Being able to compute, but not being able to solve problems is analogous to being able to read musical notation, but not being able to play a musical instrument. This point is made by Usiskin (1978):

> The crucial issue is not how the calculation is accomplished but rather knowing when and how to use arithmetic to solve problems or answer questions that in fact matter in the lives of people. Insisting that all children must be excellent pencil-and-paper calculators puts the emphasis in the wrong place - on the means, rather than on the ends, of calculation. (p. 413)

Although not a replacement for thinking and reasoning, the hand-held calculator can provide teachers with additional time to help children become numerate: i.e., to understand and interpret quantitative situations and to solve everyday problems. I believe that our school instruction should be guided by the general principle that children should not be forced to learn algorithmic procedures that can be done better and more

Table 6: Comparing Written and Calculator Computation Accuracy and Speed

Method	Accuracy (%)		Time (min)	
	Grade 7	Grade 8	Grade 7	Grade 8
Pencil-and-paper	62	68	42	34
Calculator	96	91	9	9

easily by machine. Instead, we should concentrate on teaching those things that humans do better than machines.

Unfortunately, despite their usefulness and contrary to the beliefs of the media and general public, calculators are rarely used in schools and, in those few classrooms where they are used, children often have to demonstrate a good mastery of written computation before being allowed to use a calculator. This practice seems analogous to insisting that all farmers learn to team horses before learning to drive tractors. It seems unreasonable to require children to serve a lengthy apprenticeship in written calculation before they earn the right to use a calculator.

The time has arrived for schools to take advantage of the greatest advance in portable computational aids since the Hindu-Arabic algorithms obsolesced the decimal abacus and to re-examine and revise the present unrealistic expectations for computational mastery. The present standards for year 4 students outlined in the Saskatchewan Division I and II mathematics curriculum guide provide a good illustration of the need for revision. The year 4 students are expected to be able to "subtract two numbers with minuends less than 1 000 000 with and without regrouping" and "multiply whole numbers involving a 3-digit factor by a 2-digit factor with and without regrouping." Although few teachers would assign such excessively large computations, the inclusion of these standards in a curriculum guide does provide teachers with the justification to continue to teach the computation they have always taught. (Similar standards, incidentally, can also be found in textbooks used in Saskatchewan 75 years ago.)

Because calculator use is still regarded as controversial, most teachers will not use calculators until approval has been received from their local administrators. Presently, most school boards either have no explicit policy on calculator use or, if they do, their policies are often unrealistic or fainthearted. It is important for Saskatchewan Education to develop new guidelines regarding calculator use and to ensure that administrators, teachers, school boards and the general public are aware of these guidelines.

With few exceptions such as evaluating the ability to perform simple computations manually or in the head, calculators should be allowed during testing. It is obviously hypocritical and pointless to instruct children on the use of calculators and then deny them the right to use calculators on tests. If calculator use in school becomes widespread, new arithmetic tests will have to be developed and standardized. The school boards who presently make use of standardized tests such as the Canadian Test of Basic Skills may need to reconsider the usefulness and validity of these instruments in light of the advances in calculator technology.

Saskatchewan Education should expect that a policy supporting the use of calculators in most grades will be opposed initially by some teachers, parents and school boards. It is inevitable that an inexpensive machine that takes away much of the need to learn aspects of arithmetic that have been considered vitally important for decades will be viewed with suspicion. However, the nostalgia for obsolete skills and practices cannot be allowed to dictate what should be taught in the future.

Guidelines for Calculator Use in Schools

In the elementary and middle grades, children should be provided with easy access to simple 4-function calculators. In order to assure that some degree of standardization occurs, schools should consider either purchasing classroom sets or recommending a particular calculator for purchase by parents, much in the same manner as dictionaries and atlases are presently recommended. Saskatchewan Education should consider following the practices of some states such as Conneticut who negotiated with several manufacturers to provide all schools with very inexpensive but reliable cacluators.

Solar-powered or button battery-powered LCD calculators (the thin ones with the black numerals displayed on a silver surface) are recommended because of their inexpensive operating costs - batteries can be expected to last throughout a student's elementary education or even longer if the calculator has an automatic shut-off feature. Battery-operated calculators seem to be slightly more durable than solar-powered calculators. Almost all the major manufacturers produce good and inexpensive 4-function calculators.

As children become more mathematically sophisticated, other types of calculators can be introduced. For example, every middle grades' classroom should have a few SI/Imperial conversion calculators available for student use. Students should advance to using scientific calculators incorporating exponential notation by about grade 8. These types of calculators can be used throughout secondary school, especially in mathematics courses where calculators are rapidly becoming an integral part of the program.

Secondary students should also learn to handle some features of more specialized calculators such as those designed for business and statistical analysis. The special function keys of these types of calculators should be introduced as needed.

Generally speaking, calculators should be used regularly in grades 4 onward as a problem-solving tool, treated much in the same manner as dictionaries. Dictionaries are used by the literate person to check the spelling, pronounciation, usage, origin and meaning of words. However, just as literate people do not rely entirely on a dictionary to carry out their daily communication tasks, numerate people do not rely entirely on a calculator to carry out their daily computational tasks: for straightforward computations, the figuring is done either with paper and pencil or in the head. Few educated persons would use a calculator to figure out the total of 15 dimes, to calculate the area of a 5 metre by 7 metre room, or figure the number of eggs in a carton of 2 dozen, for example. Teachers will need to ensure that students learn to distinguish between the appropriate and inappropriate uses of calculators.

Calculators hold special promise for helping children who continually have problems with computation. For example, one study (Wheeler, Ford, Nietupski, Loomis, & Brown, 1980) found that instruction in the use of pocket calculators greatly enhanced the potential for independent performance of functional skills such as shopping by adolescent moderately and severely mentally handicapped students with minimally developed math, money, and reading skills (p. 111). The major goal of arithmetic instruction in special education and remedial courses should be to ensure that all students can use a calculator confidently and intelligently to aid in the arithmetic needed in daily life.

Calculator use should be introduced in the primary years to explore number relationships. For example, grade 1 children can be led to the rule for adding 10 to a single digit calculator by using the addition constant feature of a calculator. From grade 4 onwards, children should gradually learn to use all the keys on the standard 4-function calculator. Each key function should be introduced in the context of solving a problem. For example, when investigating a problem that requires a number of different operations to be applied (e.g. finding the total of several purchases like 3 @ $19.37, 8 @ $14.54, 12 @ 23.05, etc.), the cumulative memory feature can be especially useful.

Much work will have to be done to help children interpret the answers displayed on the calculator. For example, if an average wage for several workers is calculated as 31118.333, it should be read as either $31 118.33 or rounded to values like $31 118.00 or $30 000.00. Students need to learn that, in the case of money calculations, decimal figures beyond the 2nd digit are generally ignored. Reading and approximating decimal numbers are essential requirements for using calculators intelligently.

Because of the present tendency of calculators to handle only decimal number arithmetic, problems involving fractions have to be converted to decimals (dividing the numerator by the denominator). Students should be aware that, in most cases, only a decimal approximation is possible.

Students should also learn that particularly small or large numbers cannot be computed with a calculator because of its limited memory capacity. However, it is extremely unlikely that students will ever encounter a practical problem that exceeds the capacity of the simple 4-function calculator.

Various ways to check the reasonableness of a calculation by forming an estimate or by determining if the answer makes sense in the context of a realistic problem (e.g. it makes no sense to buy 6.46 eggs for a recipe) will have to be learned. Learning to evaluate the reasonableness of an answer requires a great deal of practice and experience and teachers should expect some children to encounter difficulties. (It should be noted that one study showed that even good adult estimators were so trustful of the calculator that they did not detect errors in calculation produced by a calculator that had been re-programmed to give incorrect answers).

Because of the speed and ease of operation of the calculator, students will likely be more willing to check the accuracy of calculations done by a calculator than done by hand. Various checking techniques should be taught: e.g. using the same operation, but a different order of steps (adds 12.56, 12.85 and 12.64 and compares to the sum of 12.64, 12.85 and 12.56); using a different operation (multiplies 124.88 and 23.7 and checks by dividing the displayed quotient by 23.7 or 124.88).

Pencil-and-Paper Computation

Students still need to know some methods of written calculation. However, computational methods that are unnecessarily complex, error-prone, or time-consuming should be excised from school curricula. Instead, children should learn simple algorithms that they understand, although these algorithms need not be as efficient as the ones they are presently required to learn. For all practical purposes, practice in whole and decimal arithmetic should rarely go beyond finding the sum of several 4-digit numbers, the difference between two 4-digit numbers, the product of a 1-digit and 4-digit number, and the quotient of a 4-digit dividend and 1-digit divisor. Most of these calculations should be done while carrying out simple money transactions or working with units of measurements.

In the elementary grades, the focus should be on decimal rather than fractional arithmetic. Although computation with common fractions should still be included, complicated algorithms should be delayed until at least secondary school where they are a valuable introduction to rational algebraic expressions.

More time must be spent in school mathematics on developing an understanding of the number ideas underlying algorithms - why they work the way they do. Traditionally such understanding has been ignored by teachers as the mechanics of how to do calculations has been their primary concern. The understanding of decimal-based algorithms is best developed through the use of manipulative materials that illustrate the structure of the decimal number system: e.g. devices such as base 10 blocks, various abacuses, metre sticks, money, place value charts, bundling sticks, etc. These materials can be used throughout the elementary and middle grades. A major emphasis should be placed on the idea that computation almost always involves some form of regrouping or exchanging: e.g. in finding the difference between 123 and 59, 23 is regrouped as 1 ten and 13 ones. Exchanging is a difficult idea whose understanding develops slowly and only after much work with appropriately structured concrete manipulatives.

It is no exaggeration to claim that computation with fractions has rarely been understood by most students or even adults - how many adults can give a sound reason for inverting and multiplying when dividing two fractions, for example? Because of their abstract nature, many of the operations on fractions should be delayed until the upper middle grades or early secondary years.

As the review of several studies of everyday arithmetic indicated, more time must be spent in school on helping students learn to handle computation and to solve practical problems involving percentages. The present attempt to teach most percentage concepts in a hurried and often superficial manner in grades 6 to 8 has not been every effective. Instruction on percentages should continue throughout secondary school with special emphasis on their uses and abuses.

In the elementary and middle grades, and even in the secondary grades, much more consideration should be given to teaching intuitive and more realistic methods of computing. (Recently a grade 12 teacher told me that he had many students who, although they were succeeding in algebra and geo-trig, couldn't reason out the exercise 6 ÷ 1/8 because they had "forgotten the rule"). When people find themselves in the rare practical situations where an understanding of fractions is needed, they usually do not use the formal methods for working with fractions taught in school. Instead they usually resort to more intuitive methods such as those used in the kitchen or workshop. For example, if a whole recipe calls for 2/3 of a cup of butter, the number of whole recipes made from 4 cups of butter could be calculated by applying the conventional algorithm to the calculation 4 ÷ 2/3. However most people prefer approximations: "Since 2/3 cup is smaller than 1 cup, more than 4 whole recipes can be made from 4 cups. Likely 5 to 6 recipes can be made." Those wishing a more exact answer might use reasoning like this: "There are 3 two-thirds cups of butter in 2 cups so 4 cups would make double this or 6 recipes exactly." Some people might even simulate the problem by pouring out measured cups of water. Intuitive methods of calculation should be seen by teachers and students as having a legitimate place inside as well as outside of school.

All teachers must realize that many children as well as many adults do not understand how the formal computational procedures taught in school are connected to real-world problem-solving. There is recent evidence that when calculation needs to be done outside of school to solve some practical problem, even children prefer informal methods of calculation over school-learned procedures.

In one study (Carraher, Carraher & Schliemann, 1987), the ability of young Brazilian "street children" to solve simple story problems and to carry out corresponding written calculation exercises was investigated. Although the children could usually solve the problem using an informal computation procedure, they often could not carry out the school-learned procedure. For example, when given the exercise 100 ÷ 4, one youngster attempted to divide 1 by 4, which she concluded was not possible, then tried to divide 0 by 4, and finally gave up. When asked to figure the answer using any method she desired, she said, "See, in my head I can do it. One hundred divided by four is twenty-five. Divide by two, that's fifty. Then divide again by two, that's twenty-five" (p. 93). In the elementary and middle grades, intuitive methods of calculating and reasoning with numbers as they are applied to the solution of realistic problems should be encouraged.

Mental Calculation and Estimation

Despite the obvious advantages that electronic calculators hold over all calculation methods, the ability to calculate mentally and to provide quick estimates of the accuracy of a calculation, whether produced electronically or manually, remains a useful tool for handling the many commonly seen tasks of consumers and workers. For example, consider how the following routine problems would be handled outside of school:

1. A purchase of $1.83 is made and the cashier is handed a $5 bill. How much change should be received?

2. The departure board at the airport shows that a flight is scheduled to leave at 8:35. Is there enough time for a snack, if the present time is 7:40?

3. A committee meets every second Monday of a month. If a meeting is held on January 13, what will be the scheduled date of the next meeting?

4. A jogger finds that his heart beat 39 times in 15 seconds. What is his heart rate in beats per minute?

5. A restaurant bill is $59.88. Using 15% as an acceptable service charge, what amount should be left as a tip for the waiter?

6. If a recipe calls for 1/2 cups of butter, how much is needed for 6 full recipes?

Although the need for accurate answers would vary from problem to problem, chances are the numerate person would mentally calculate the solution of each problem, especially if a pencil and paper or a calculator were unavailable or too inconvenient to use.

Learning to calculate mentally is an important life-skill because the great majority of everyday problems are solved mentally. Furthermore, the intelligent use of a calculator requires a sound knowledge of estimation techniques in order to check the reasonableness of a displayed solution and mental calculation is the cornerstone of all estimation processes. For these practical reasons, children should be taught a variety of ways to calculate in the head.

Unfortunately, pencil-and-paper figuring has so dominated our elementary school mathematics program that instruction in mental calculation has been neglected. Perhaps because of this neglect, most people are not proficient mental calculators. A colleague of mine related this not too atypical ancedote. He wanted to buy some Halloween candies selling for a price of 3 packages for $1.00. To make it easier for the cashier - or so he thought - he bought 39 packages. When he took his purchase to the cashier, she asked him why he wished to buy 39 instead of 40 packages. He explained that since 39 is divisible by 3, but 40 is not, the price calculation is simplified. Instead of performing a quick mental calculation, the cashier took a pencil and paper and labouriously divided 39 by 3. During later conversation, she commented that she was looking for a job more suited to her university qualifications.

Judging from the results of many studies such as the United States Third National Mathematics Assessment (NAEP, 1983) and the ACACE study discussed in Part 2, this young employee's inability to free herself from written methods of calculation is not an isolated case. A large number of children and young adults cannot perform even the most straightforward mental calculations.

Table 7 summarizes some of the results of the National Assessment testing. It is interesting to compare the results of calculations done mentally with those done with paper and pencil. After being presented with a computation question, the 13-year-old students were asked to identify their preferred method of solution: mental, paper and pencil or calculator. The results summarized in Table 8 suggests that many children are enslaved to the written procedures taught in school.

Learning to calculate mentally and to produce estimates can also help in the development of, what can be described as, a keen "number sense." In contrast to the traditional written methods of computation that children often do mechanically and without a great deal of thinking, mental calculation is done usually by using some apprehended number property or pattern to expidite the solution to a calculation problem. This emphasis

Table 7. Selected mental and written computation questions from the United States Third National Mathematics Assessment.

	Mental calculations		Written calculations	
Age	Question	% correct	Question	% correct
13-year-olds	480/16	40	2-digit division	80
	3500/35	40		
17-year-olds	90 x 70	55	2-digit multiplication	90
	8 x 625	45		
	480/16	60	2-digit division	80
	3500/35	60		

Table 8. Computational preferences of 13-year-olds for selected problems on the United States Third National Mathematics Assessment.

	% preferring a method			
Computation	Do in my head	Use paper and pencil	Use a calculator	No response
4 x 99	44	39	16	1
945 x 1000	38	31	31	
$40\overline{)2800}$	31	45	23	1

on thinking about numbers and number relationships rather than about digits and bookkeeping rules so prevalent in written arithmetic contributes to an increased number sense. A well-developed number sense ensures that a person will not have to turn to a machine or pencil and paper for every straightforward calculation encountered in daily life.

Despite the recognized benefits of learning to calculate mentally, few teachers provide children with explicit instruction and regular practice. A lack of emphasis on mental arithmetic and a consequent over-emphasis on formal written methods of calculation is perhaps not surprising because curriculum developers and textbook writers have not identified and sequenced the various ways that calculations can be handled mentally. Any subsequent revision of the mathematics program should include a listing of the important mental methods and their corresponding practical uses.

Regardless of the mental methods chosen to be taught, a solid base of understanding should be developed before expecting children to have some facility with these methods. Teaching a number "trick" by rote defeats one of the purposes of calculating mentally. Concrete objects, number patterns and analogies are suitable starting points when trying to develop a calculating strategy.

For example, many mental calculations can be based on a students' familiarity with money. Primary children who struggle to learn difficult addition facts such as adding 9 can use dimes and pennies to develop the following reasoning: "To add 5 cents and 9 cents more, add a dime instead of 9 cents to get 15 cents and then subtract 1 cent." Working with quarters and dollars can provide the background knowledge to help older children learn the useful mental multiplication strategy, aliquot parts:

e.g. To multiply 16 x 25, think 4 quarters = $1.00 or 100 cents; 8 quarters = $2.00 or 200 cents; 12 quarters = $3.00 or 300 cents; 16 quarters = $4.00 or 400 cents; therefore 16 x 25 = 400. Research has demonstrated that such number reasoning is helpful in learning the number tables by heart (Baroody, 1985; Fuson, 1986; Steinberg, 1985; Thornton, Jones, & Toohey, 1983). Likely there is a reciprocal relationship between learning to calculate mentally and remembering specific number combinations.

When the students appear to understand the number principles and relationships underlying a mental calculation technique, a more efficient abridgement should be introduced and practised. In the case of the adding 9 strategy, students can think "add 10, take away 1," while multiplying by 25 can be reduced to reasoning "Divide by 4, multiply by 100" or this reasoning can be even further abbreviated to "Take 1/4 and then annex ('add') two zeroes."

Once the students understand how and when to apply a mental calculation technique, practice is needed to increase speed and accuracy. To develop flexibility in thinking about numbers and number relationships, children should be encouraged to suggest alternative and possibly more creative ways to solve each practice exercise. A 2-3 minute oral "warm-up" preceding most mathematics lessons should provide enough practice without detracting from other regularly scheduled lessons. Whenever possible, the practice exercises should illustrate some real-life problems. They should be presented orally and with the expectation that answers would be reasoned mentally: e.g. (1) How much is a quarter, a nickel and 3 pennies? (2) How many buns in 5 dozen? (3) How many kilometres in 1234 m? (4) In a half price sale, what would a regularly priced item of $5.60 cost? (5) How much change from $10 would you receive for a $2.95 purchase?

It is very important to encourage children to use mental calculation and estimation whenever the opportunity arises in a regular arithmetic lesson as well as in lessons in other subject areas. Students should be invited to bring examples from magazines, newspapers, and advertising flyers to illustrate some real-world uses of mental calculation and estimation.

Children should not be discouraged from using short-cut mental methods to eliminate steps in performing written calculations assigned in regular arithmetic classes: They should be challenged instead to try to use a mental calculation to solve all or part of a calculation problem before reaching for a pencil and paper or calculator. Unfortunately, many good mental calculators lose their figuring skills and associated number sense when teachers continually force them to "show all their work."

Some reasonable time limit is suggested for all practice and testing activities involving mental calculations because studies have found that children often resort to pencil and paper computations if given too much time to complete a set of calculations. The most appropriate timing standard depends on the ability of the students and the complexity of the calculations.

Undoubtedly, children will vary in their ability to calculate mentally. The more able calculators need to be challenged to extend a strategy to more difficult calculations. For example, a difficult calculation such as 25 x 49 can be done in a variety of ways: (1) Think 25 x 48 = 1/4 of 48 x 100 = 1200 plus another 25 equals 1225; (2) Think 25 x 50 = 1/2 of 25 x 100 = 12.5 x 100 = 1250 less 25 equals 1225. The less able calculators should be encouraged to calculate in the head with simple numbers: 8 x 25, 7 x 80, 100 + 500, etc. When difficult mental calculations are attempted, the less able should be encouraged to produce at least a good estimate.

Although estimation can be thought of as calculating mentally with approximate rather than exact numbers, children need to learn how to adapt a mental calculation to work with approximate numbers. Such instruction should begin as soon as children learn to operate on numbers and have committed some number combinations to memory. For example, primary children can be encouraged to estimate sums of money like 25 cents and another 27 cents by using any of the following methods: (1) by rounding and adding 30 cents and 30 cents; by working on only the 'front-end' digits and adding 20 cents and 20 cents; by revising the front-end estimate of 40 cents by adding another 10 cents because 5 cents and 7 cents is slightly more than a dime. The astute primary child may even prefer to calculate an exact answer by reasoning "25 cents and 25 cents makes 50 cents and another 2 cents makes 52 cents."

Middle grades' and secondary students should learn how to produce estimates with decimals, fractions and percents. For these types of estimations, a good understanding of the relative size of such quantities as well as a knowledge of how to convert from one form of number to another are needed. For example, regardless of the way a discount of 35% of an item priced at $23.85 is estimated, much understanding about numbers is required: e.g. (1) reformulate as 30% of $20.00, consider 1/10 of $20.00 = $2.00, multiply by 3 to get an estimate of $6.00, and finally refine by increasing estimate slightly to $7.00; (2) repeat the preceding procedure, but round to $24.00 to yield an estimate of $7.20; (3) reformulate as 33 1/3% of $24.00, consider 1/3 of $24.00 to get an estimate of $8.00; (4) reformulate as 35% of $24, calculate 10% of $24, then triple this amount and add one half again.

These examples show that a considerable amount of reasoning is required to produce a good estimate. One difficult aspect of estimating is understanding that the precision of an estimate will always be governed by the context and the ultimate use of a calculated answer. This difficulty is often not appreciated by adults, especially those who criticize young people for their apparent lack of number sense. Children need considerable help and practice if they are to learn to distinguish between situations calling for rough estimates, fine estimates or exact answers. This type of discrimination is neither easy to teach nor easy to learn because there are no clear rules to follow and many subjective decisions must be made instead. However, all subject areas can contribute greatly to the development of a good number sense by encouraging children to estimate computations and to discuss the ways in which their estimates were produced.

Since many teachers are not familiar with the various ways that numbers can be mentally calculated and estimates can be produced, the following sources will prove helpful:

1. Mental Math in the Middle Grades is a teaching program of 36 lessons on mental calculation published by Dale Seymour publishing and distributed in Canada by Spectrum and GLC. Step-by-step lessons on blackline masters can be presented as overhead transparencies or as handouts. Practice exercises, tests, and teaching suggestions are also included. Companion books for the primary and junior high grades are currently being developed. The junior high book is expected to be available in the fall of 1987, while the primary book will likely be

published in the spring of 1988.

2. Computation Estimation is a series of 3 books for grades 6, 7, and 8 each containing 15 lessons on various computational estimation techniques published by Dale Seymour and available from Spectrum and GLC. The series is based on the lessons used in a recently successful instructional study sponsored by the NSF.

3. Estimation and Mental Computation is a collection of excellent articles prepared by the National Council of Teachers of Mathematics related to teaching estimation and mental calculation. Every school library should have a copy for teachers.

4. The Arithmetic Teacher has begun to feature a section of teaching ideas for estimation and mental computation. This journal is published by the National Council of Teachers of Mathematics and is probably the best single source of ideas for teaching elementary and middle grade mathematics available.

Measurement

Studies of the everyday activities of the consumer and worker demonstrate clearly that a functional knowledge of measurement is extremely important, especially in this age where the tendency is to measure everything and to deny the existence of those things that cannot be measured easily. Although many measurement ideas are learned incidentally by participating in life experiences, the school has a responsibility to ensure that students develop an understanding of the nature of measurement as a system as well as acquire specific knowledge about the commonly seen measurement processes of length, area, volume, capacity, temperature, time, mass (weight), and angles.

Measurement is a way of quantifying either the properties of objects or their states. Hence, it must always be clear to children exactly what characteristic is being measured. Teachers can help children perceive measureable attributes by using appropriate vocabulary and by providing direct experiences related to a measurement idea: e.g. lifting various objects to feel their differing weights; pouring water into different sized containers to see their differing volumes; covering different sized regions to see their differing areas.

In the initial stages of concept development, the vocabulary should be informal and there should be no attempt to use either standard units or conventional measuring instruments. For example, area should be considered as a "covering" of surfaces, while volume should be considered as a "filling" of space. If children have an intuitive notion about "what" they are measuring, they will be able to distinguish between similar appearing measurement processes and be less likely to confuse, for example, surface area with volume, a common occurence among students and adults.

Once the intuitive ideas about measurement have been explored adequately, children should learn how to compare objects to determine either their equivalence or their non-equivalence. Non-equivalence can be easily demonstrated in those instances where an attribute of object A can somehow be considered as "part" of the attribute of object B. In these cases, the measure of object A can be said to be less than the measure of object B and, conversely, the measure of object B can be considered to be greater than the measure of object A: e.g. a book covers less area than the table on which it is resting. Equivalence of measures exists when an attribute of object A can somehow be considered congruent or identical to an attribute of object B. In this case, both objects will have the same measure. These are important principles that allow the relative measures of objects to be determined without having to resort to the inconvenience of assigning a number or using a measurement tool.

When most people use direct comparison of objects rather than a measuring tool to determine equivalence and non-equivalence, it is usually applied automatically: e.g. children are noted to be shorter than their parents who in turn are both noted to be shorter than any doorway in the house. In none of these cases is it necessary to measure height or refer to a numerical value. Learning to compare and order objects enables a child to develop an understanding of relationships such as "shorter than", "heavier than," "warmer than", "fuller than," etc. Many experiences involving comparisons and their ordering should be provided before proceeding with any instruction involving standard units and the use of conventional measurement tools.

Students have to learn eventually the quantification procedures used to assign numbers to an attribute or state of an object. In order to focus on the understanding of the procedure, non-standard units should be used initially. For example, the area of any region can be determined by counting the number of unit areas needed to completely cover the region. Children can learn to calculate area initially by using such non-standard units areas as hands, books, blocks and pennies, for example, before proceeding to use standard unit areas like cm^2 or m^2. For measuring time, a very abstract and poorly understood notion, children should measure the duration of simple events initially in terms of handclaps, water drips, ball bounces and pendulum swings before using a clock calibrated in seconds, minutes or hours.

Students of all ages need experiences in carrying out measurements using a variety of conventional measurement tools. This active experience is often absent in textbook- or workbook-dominated classrooms where the emphasis is placed on learning about units of measurement. Providing students with extensive practical work ensures that they will have less difficulty in learning to use tools and instruments encountered outside of school.

Whenever measuring takes place in any subject area, it should be done primarily for the purpose of gathering data needed for the solution of a problem or the interpretation of a quantitative situation. With the exception of the introductory lessons on using a measurement tool, students should not be measuring merely for the sake of measuring.

Since each type of measurement tool has its own idiosyncratic features, children should be given experience with a wide variety of measurement tools: A rough rule of thumb to follow is "the more types of measurement tools seen and used the better." Students should learn to measure length using rulers, metresticks, cloth and metal tapes, trundle wheels (both those used for long distances and those used for map work), calipers (vernier, bow), micrometer and feeler guages. Area should be measured using unit squares, graphing paper and transparent grids. Volume and capacity should be measured using unit cubes, standardized spoons and cups and graduated beakers. Mass (weight) should be measured using balance pans (equal-pan and one-piece) and standard weights, compression scales (kitchen and bathroom) and a variety of spring balances.

Students should learn that tool selection is guided by the demands of the particular measurement situation. For example, a 30-cm ruler is well suited for measuring small straightline distances, but it is highly unsuited for measuring short or long distances and the lengths of curved objects. Students should also learn how to reason with measurement tools and apply what can be called "the tricks of the trade": e.g. measuring length with a ruler whose initial marks are either missing or illegible; measuring the inside dimensions of a wall using a metal tape by reading the length of the exposed tape and then adding on the dimensions of the case; measuring the circumference of a sphere by wrapping string around the sphere and then measuring the string against a ruler; measuring the thickness of a pipe by subtracting the inside diameter from the outside diameter and then dividing by two; dividing a length of board into 4 equal pieces. Teachers should remember that this type of "folk knowledge" is best learned through practical experiences rather than through chalkboard demonstrations.

Children also need to be provided with ample practice in reading a wide variety of instruments and scales frequently seen outside of school: e.g. thermostats, oven temperature dials, timers, parking meters, gasoline pumps, etc. Particular emphasis should be on reading analog instruments because of the difficulty in determining the scale, which is often not made explicit. Children still need to learn to tell time. Experience with both types of clock faces should be provided. Because children experience difficulties with some conventions of the traditional analog clockface, mastery in telling time should not be expected until the upper elementary or middle grades.

As students become proficient with measuring tools, they should learn to appreciate the inexactness of all measurements regardless of the care taken in measuring or in the choice of instrument used to carry out the measurement. Approximation is an inherent characteristic of the measurement of any continuous quantity or attribute. The inexact nature of measuring is an idea that is difficult to teach because most people believe that our physical measurements can be absolutely accurate.

As a first step in developing the notion that measurement is approximate, students should learn that, in measuring continuous quantities, upper and lower limits have to be established: "This container holds more than 5 cups but less than 6"; this pencil is more than 15 cm long but less than 16." Much work should be done with students to help them use measurement tools to determine boundaries and to estimate when a measure falls between these two boundaries. As they become more experienced, students should understand that measuring requires an observational decision to specify the closer unit of a bounded measure: for example, if the measure appears to be closer to the 8 cm mark than the 9 cm mark, then 8 cm is specified as the length.

As the need arises to improve the quality of measurements, students can be introduced to more precise tools containing sub-divisions of units: measuring with a ruler divided into millimetres rather than centimetres. Students should learn that more precise instruments are not necessarily superior to less precise instruments. In following a recipe, measuring volume in "pinches" is quite adequate, but "pinches" are clearly inadequate in filling a drug prescription. They should learn that standards of accuracy and precision are subjective decisions guided by practical circumstances.

For most students, there is little need to learn the calculational procedures for determining accuracy, relative error, and the number of significant figures. Too often these procedures are introduced in science and mathematics courses on a formal, definitional basis without consideration for how accurate the measurements need to be to fit the task. Students need to know, however, that it is misleading to express measurement calculations in units more precise than the original measurements. For example, if three measurements are taken as 105 mm, 104 mm, and 104 mm, the average reading should be specified as 104 mm, not as 104.33 mm because the precision of the instrument used to gather the measurements was calibrated in units of 1 mm not 0.01 mm.

The concept of error of measurement is rarely treated realistically in mathematics courses. Most texts, and many teachers, do not bother with error factors in computation except in those cases where the text includes a section on accuracy and precision. Consequently, many students dismiss error considerations as unimportant. Whenever real measurement takes place in school, there should be some discussion about precision and error and their effects on computation.

As the review of the literature in Part 3 indicated, perhaps the most significant contribution schools can make to a student's understanding about the nature of measurement is to ensure that a "feel" for measurement is acquired. When people have an appreciation of the size of units and their relation to everyday objects, they know instinctively when a measurement statement seems sensible or not: e.g. realizing that a calculation of 100 metres for a length of board is unrealistically high. If a feel for measurement is not acquired, then almost any figure is apt to be accepted as being reasonable. A feel for measurement is becoming increasingly essential for functioning in this technological world where people sometimes blindly accept whatever figure is offered up by a machine.

A good sense of measurement develops slowly. The 1977 British Columbia Mathematics Assessment (Robitaille & Sherrill) found, for example, that only about 25% of 10-year-olds were able to select 35 kilograms as an appropriate mass for a 10-year-old boy given the choices of 35 grams, 35 kilograms, 75 grams, and 75 kilograms (p. 46). The majority of students selected the "Tom Thumb" estimates of 35 grams and 75 grams. Four years later, only 20% of grade fours made the correct selection (Robitaille, 1981). After a decade of the exclusive use of SI units in schools, it is hoped that students now have a better feel for this measurement system.

Essential to the development of a feel for measurement is the establishment of a wide variety of everyday equivalences that can be used whenever measurement units are being considered: e.g. doorways are about 2 metres in height; the mass of packaged butter is 500 grams or 0.5 kilograms; the width of a dime is about 1 millimetre; the distance from Saskatoon to Yorkton is around 300 kilometres; milk cartons are usually 1 or 2 litres in capacity. Whenever measurement statements are seen in school subjects, teachers should ensure that students can translate these statements into statements involving equivalences about everyday objects or situations. Prompting questions should be used to elicit what can be described as, "bigger-than-a-bread-box thinking": e.g. Would a length of 10 metres be higher than the doorway? Does the answer seem sensible in light of this fact?

Most subject areas can contribute significantly to the development of a measurement sense. Whenever, a geographical area is being discussed in Social Studies, for example, its size should be compared to sizes understood by the students: making comparisons with the sizes of Saskatoon, Saskatchewan and Canada. Charts portraying the relative sizes (e.g. a sketch of Germany placed on a map of Saskatchewan) should be displayed prominently in the classroom.

In the physical sciences, where measurement is commonplace but the units are not, teachers have to make a special effort to ensure that quantitative statements are understood: Do students really have a feel for units such as light years, joules, newtons, milliamperes, and nanometres? Can they identify unreasonable statements involving these units? For example, from personal experience I know of adults who have no idea whether or not 1.5 volts is a harmfully high voltage, although they usually know that flashlight batteries can be handled safely. Certainly there is no evidence that learning to compute with these abstract quantities contributes significantly in the development of a measurement sense.

Students should also be able to use their sense of measurement to provide crude estimates: "The length of the classroom is between 10 and 15 metres." Initially, estimation should involve the presence of objects that can be used as a referent: "About how many of these metre sticks would it take to reach the ceiling?" As students become more experienced in estimating and can visualize a measurement unit, both the object to be estimated and a useful referent unit should be physically absent. However, it must be emphasised that a high degree of accuracy in estimation develops slowly, if at all, and mainly through continual usage and much practical experience rather than through instruction provided in school. For example, the ability of mechanics to discriminate between different but similar looking sizes of wrenches, sockets, nuts and bolts undoubtedly develops mainly on the job rather than in school. Nevertheless, throughout their school years, students should be encouraged to practise making rough estimates of lengths, areas, volumes, capacities, masses, temperatures and angles.

Students still need to learn some useful ways for determining the area and volume of commonly seen objects: e.g. calculating the area of a rectangle, triangle, parallelogram, etc. However the usual practice of forcing students to memorize formulas for every specific instance should be delayed until students are provided with sufficient informal measurement experiences. Otherwise, when students are forced to memorize formulas they don't understand, they often either confuse the formulas with each other, misapply them, forget the meaning of the terms (e.g. forgetting how to determine b and h in the recalled formula $A = 1/2\ b \times h$), or approach a non-routine problem in a very mechanical manner. The inability of students to reason out a non-routine area problem was demonstrated by grade 12 students in the 1977 British Columbia Mathematics Assessment (Robitaille & Sherrill) when they were asked to compute and select from a number of choices the area of a figure formed by two concentric rectangles. This is a relatively straightforward problem requiring the subtraction of the areas formed by the inner and outer rectangular regions, yet only about half selected the correct response. Most students indicated they either didn't know how to proceed or selected the area of the inner rectangular figure as their choice (p. 138).

Instead of providing extensive practice with plugging numbers into formulas, students should be helped to reason out area and volume problems, usually by rearranging a complex shape into another less complex shape whose properties are known or by subdividing a shape into a series of simpler shapes. For example, the area of a triangle can be reasoned as one-half of the area of the parallelogram formed by the addition of another congruent triangle and the area of a parallelogram can be reasoned as being equivalent to the area of the rectangle formed by removing one triangular section from one end and placing it on the other end. Whenever problems involving indirect measurement are given to students, the emphasis should be on reasoning rather than on applying a formula mechanically.

All measurements and measuring tools seen in school should continue to involve SI (metric) units, especially in the elementary and middle grades. It is the responsibility of all subject areas to help children to think and to estimate with SI units. Unfortunately, due to the reluctance of some older adults to learn the fundamentals of the SI system, the use of Imperial measures in the U.S.A., the continued use of some types of equipment based on Imperial measures, and, most importantly the recent changes in the federal legislation allowing the use of both SI and Imperial measures, the change from the Imperial to the SI system of measurement has not proceeded as quickly as had been expected, although considerable progress has been made, and students will likely have to contend with some types of Imperial measures for many years to come. It is the continued use of Imperial measures outside of school that is proving to be the greatest difficulty for schools, not the teaching of the relatively easily mastered SI system.

With some reluctance, it is recommended that Saskatchewan Education consider sponsoring a study to determine if a familiarity of Imperial measures is still required by employers and to ascertain if a lack of familiarity by graduating students is a cause for concern for either employers or employees. The study should also determine whether the SI system is used exclusively in all subject areas and by all teachers.

In the meantime, Saskatchewan Education should continue with its present policy of promoting the exclusive use of SI in the elementary and middle grades years. Judging from the results of the recent B.C. Mathematics Assessment, all teachers need to spend more time on teaching students to convert from one SI measurement unit to another: e.g. changing 14.7 cm to 147 mm.

In the secondary years, students should continue to use the SI system, but if there is convincing evidence that the exclusive use of SI in schools is causing difficulties for its graduates, then perhaps an attempt should be made to ensure all students are familiar with the more common Imperial units such as feet, inches, yards, pounds, ounces, pints, gallons and acres and are able to use any commonly seen measuring instruments calibrated in Imperial units.

If Imperial measures are presented in the secondary years, there should be no attempt to have students learn conversion factors by heart (e.g. 1 metre = 39.37 inches, 1 kilogram = 2.2 pounds). Instead, Imperial units should be studied in relation to their SI counterparts: e.g. 1 yard is slightly less in length than 1 metre. Rough equivalences should be introduced when conversions are necessary: e.g. "Since 1 Imperial gallon has a capacity between 4 and 5 litres, 60 litres is equivalent to between 12 and 15 gallons. For those rare occasions where more exact conversions are needed (e.g. changing a carpet area calculation to square yards from square metres or converting cubic yards of gravel to cubic metres), tables of conversion factors, lists of equivalent measures, and SI/Imperial calculators should be used. There is no need to spend much time, if any, on calculating with Imperial units.

Space and Shape

Although it is important for students to learn the terminology used to describe geometric shapes (diagonol, perimeter, triangle, square, radius, diameter, etc.), the study of space and shape should go beyond the usual fare of "figure-naming" exercises so often seen in elementary school. Instead, the focus of instruction should be on the properties of plane and solid objects and how such knowledge can be applied to real-world situations.

Spatial concepts develop best through active experiences. Students should learn a variety of ways that plane and solid objects can be constructed from simple materials or drawn with conventional instruments. All students should learn to use drawing instruments such as ruler, compass, set square, and protractor. Plane and solid shapes should be available that pupils can handle and measure, and use in building.

Students should see how a knowledge of measurement procedures, construction techniques and properties of shapes can be used to carry out practical tasks such as those illustrated in the following list: (1) Using a ruler to divide a board into a number of equal pieces; (2) Using a compass or protractor to divide an angle into a number of equal parts; (3) Using a right-angled triangle formed by string with sides in the ratio 3:4:5 to square a corner; (4) Using a compass and ruler to divide a line into any number of equal segments; (5) Using a set square to find accurately the center of a circular shape; (6) Using a protractor, compass and straightedge to construct a pie graph; (7) Using a carpenter's square to bisect an angle or divide a line into a number of equal parts.

It is interesting to note that the 1985 British Columbia Mathematics Assessment (Robitaille & O'Shea) reported that, although most grade 7 students indicated they owned their own rulers and protractors, less than half reported using them in school to measure distances or angles. Such a lack of use in school of measuring instruments and construction tools probably explains why the grade 7 students had difficulties using a protractor: e.g. 50% could measure an angle to the nearest degree; 60% could draw an acute angle given its measure (p. 34). Obviously, students need more instruction and practice in using these types of simple construction instruments.

Students should also learn how to read a variety of scale drawings and plans, make reasonably accurate sketches of simple or complex structures, and occasionally construct a full or scale model using a drawing or plan. These practical activities can vary from building rectangular "houses" with construction blocks, constructing maps of the immediate area or school grounds, using a simple blueprint to construct a cardboard model, using a pattern to make some apparel of clothing, or using shop tools to build a useful object. Such activities can help students develop good hand-eye coordination, learn spatial concepts and relationships, use appropriate terminology, acquire the ability to discriminate between shapes and improve their ability to visualize.

Data Analysis and Interpretation

Knowing how to read and interpret quantitative information presented in any number of forms is essential for functioning in an information society. As a minimum, students need a facility with our base ten number system and an understanding of how symbols can be used to represent quantities. The emphasis in all subject areas should be on handling numbers seen in practical tasks: reading prices, writing cheques, reading a calculator display, using an instruction manual, following a recipe, etc.

The understanding of decimal place value relationships is best developed in the elementary and middle grades' mathematics programs through the use of structured concrete materials such as base 10 arithmetic blocks, bundling sticks, place value charts and containers, abacuses (including Chinese or Japanese), money and SI measuring instruments.

Teachers in all subject areas should be aware that children need much more experience with the "oral" number names than has been provided previously. Studies have shown that many children who can calculate the total of several multi-digit numbers cannot necessarily name the resulting sum.

Most of the work in handling decimal numbers should be restricted to 1- to 4-digit numbers (1234, 1.234, 0.1234, etc.) because studies have shown that most number work in real-life involves relatively small quantities. However, students should know how to use symbols to represent larger and smaller powers of 10: specifically, millions, billions, millionths.

Because most children (and adults, for that matter) have only a vague notion of the size of larger and smaller numbers, these numbers should be placed in a meaningful context. For example, it is easier to fathom a budget deficit of one billion dollars by considering that it is equivalent to: (1) a length of one dollar bills laid end to end 4 times the circumference of the earth, (2) an area of 4 sections of land covered with one dollar bills, or (3) a per capita debt for every man, woman and child in Saskatchewan of $1000 each. As a general principle, whenever any large or small quantities are seen in any school subject, teachers should ensure that some reference is made to everyday equivalences familar to students. Otherwise, arguments based on meaningless quantitative information will neither be understood nor appreciated.

An important aspect of data analysis is the reading and interpretation of information organized in tables and charts or displayed as graphs. To develop a facility with reading these type of data presentations, it is important that almost all school subjects contain some instruction on reading and using tables, charts, and graphs commonly seen in a particular area of study. Much of this learning should take place naturally by presenting students with real-world examples: telephone, electrical and other utility rate tables; taxation tables; tables, charts and graphs taken from Statistics Canada and other types of government publications; polling and survey results.

Students should learn how a re-organization of data can be used to influence and, in some cases, deliberately mislead the unsuspecting or trusting reader. The most common technique is to change the scale of one axis of a line graph so that relatively small increases are exaggerated and, consequently, a growth over a period of time can appear to be more rapid than it really is. Darrell Huff (1954) in his classic book How to Lie with Statistics, which should be recommended reading for all teachers, provides many examples of how an unscrupulous person or group can try to influence public opinion by carefully managing the way quantitative information is presented.

Students should be given many opportunities to collect, organize, tabulate and represent data obtained either by experimenting, surveying or by referring to publications of statistical information. For example, data gathering by elementary and middle grades' students on TV viewing habits can be collected and tabulated and used to answer such questions as "Do older children watch more TV than younger children?" Students are usually greatly interested in analysing data they have collected and in answering questions they have posed.

Little time should be spent on constructing graphs, as rarely are people required to produce graphs. However, if a suitable computer plotting program is available, students should be given some instruction on its use. Most graph construction should involve no more than the production of simple sketches.

Politicians, businesspersons, bureaucrats, media people and advertisers commonly assume that the public understands the many statements and arguments based on quantitative jargon. However, it is clear from the ACACE study discussed in Part 2 that terms like inflation are often widely misunderstood. Few people understand that a drop in inflation will not be followed necessarily by a drop in prices or that an increase in the CPI does not need to be offset by a corresponding increase in salary in order to maintain the same standard of living.

All school subjects should try to ensure that students know the meaning of the quantitative terms and indices commonly seen in everyday life (especially those reported frequently by the media), know how they are measured, and know their uses, abuses, and limitations: e.g. GNP, CPI, inflation, credit, deficit, unemployment rate, etc. In each subject area, an attempt should be made to identify the quantitative terms specific to that area of study and to ensure that students understand their meanings and interpretations.

Reasoning with Statistics

One of the more important areas of mathematics utilized in our technological society is statistics, which refers broadly to the collection, analysis and interpretation of numerical data. Because of its importance, students should become familiar with rudimentary statistical notions and be encouraged to examine critically any statistics cited as evidence for an argument or used to express a point of view. Because statistics is essentially a practical subject, students should learn when it is appropriate to collect statistical information and the reasons for doing so, the legitimate ways in which data can be manipulated mathematically, and the dangers of extrapolation and generalization.

The study of statistics should not be confined to the mathematics program, where it often becomes dry and too computationally-oriented, and fails to show the power and nature of statistics. Instead, all subject areas should provide students with practical examples of statistical analysis with the objective of developing the qualities expressed in a submission received by the Cockroft report:

> Statistical numeracy requires a feel for numbers, an appreciation of appropriate levels of accuracy, the making of sensible estimates, a commonsense approach to the use of data in supporting an argument, the awareness of the variety of interpretation of figures, and a judicious understanding of widely used concepts such as means and percentages. All these are part of everyday living. Good statistical teaching can encourage pupils to think in these ways. (1983, p. 236)

One of the most important qualities that can be passed on to students is the withholding of judgements about any argument based on statistical information until it is known how this information was obtained. As Weaver argues: "It is silly and irresponsible to uncritically either dismiss or accept arguments based on numbers; neutrality is no longer a tenable position; students need to be at least functionally literate with statistics" (1982, p. 289). Unfortunately, the magic and appeal of numbers is often so great that it brings about a suspension of common sense. Most of us are far too trusting of statistical presentations and we often substitute arithmetic for good judgement. There are several real-world examples to illustrate how most of us are far too trusting of statistical presentations and rarely make an effort to go beyond the arithmetic.

A most recent example of an obviously exaggerated statistic was reported recently by the media. Sociologist Linda MacLeod, in a study for the Canadian Advisory Council on the Status of Women, estimated that nearly a million Canadian women are abused by their partners each year: a statistic that compared favourably with her 1980 report estimating that 1 in every 10 women was abused. This is very sensational information, which has been regarded as controversial primarily because her broad definition of battering includes emotional as well as physical abuse: men who shout at their partners or who mistreat the family pet in the presence of their partners are considered to be abusive. The definition is not only far too expansive, but information required to quantify abuse would be very difficult to obtain: For example, how can the vague notion "emotional abuse" be operationally defined and properly measured? Students should understand that a great amount of subjectivity usually underlies supposedly objective statistics reported in the media.

Students should learn how sampling is used to draw inferences about a population. Real-world examples can be introduced in most subject areas: Health (drug testing, diet effectiveness), Science (wildlife population studies, pesticide testing, environmental studies), Practical and Applied Arts (product quality assessments), Social Studies (census surveys, opinion polls), Physical Education (life-style assessments, exercise research).

Students should learn the importance of using a representative and sufficiently large sample and understand the consequences of selecting a biased sample. For example, surveys conducted by newspapers, radio and TV stations are often biased because they are usually based on the opinions of readers, listeners and viewers, and the opinions of those who don't read the specific newspaper, listen to that specific radio station, or view that particular TV station are ignored. As well, the gathered statistics are based entirely on the views of those who are sufficiently interested to express these views.

Students should also learn that it is not always easy to recognise invisible sources of bias that render a sample unrepresentative of a population. One of the most famous examples of invisible bias was seen in the Literary Digest's prediction of the 1936 American election where Alfred E. Landon was predicted to defeat Franklin D. Roosevelt. The magazine's sample size was truly impressive: over 2 million questionnaires were completed. Despite the massive size of the poll, it failed to predict a Roosevelt victory, being off the mark by a substantial 19 percentage points. Two other polls, the Gallup poll and the Roger poll, did predict accurately a Roosevelt victory, however. The bias of the sample chosen by the magazine was not detected until the election results showed that the poll's predictions were embarrasingly wrong. Apparently, the Digest's sample of voters was drawn from lists of automobile and telephone owners. People who could afford telephones and automobiles in 1936 were not a representative cross section of voters but predominantly Republican. In contrast, the Gallup and Roper polls selected their samples from a proper proportion of voters from each economic stratus and were, therefore, more representative of the voting population (Gallup, 1972).

Whenever surveys and polling results are presented, students should be taught to examine carefully the procedures used to select a sample, especially those used in studies sponsored and conducted by special interest groups. Recognizing bias is an essential component of statistical thinking that needs time and practice to develop.

Basing conclusions on an inadequately small sample size is another commonly seen misuse of statistics. Although a great deal of experience and technical skill is often needed to determine the adequacy of a sample size, students can learn to identify obviously too small samples. For example, if 10 people are surveyed about their preference of two brands of soft drink and 6 choose brand A, it is obviously incorrect to claim that about 6 out of every 10 people in the population prefer brand A. By drawing at random from a container slips of paper on which the two names of the soft drinks have been written, students can learn that it is quite possible to have 6 successes out of 10 trials even if there is no real difference in people's taste preferences.

The proposed federal legislation requiring greater disclosure of polling details, although criticised by the media as being impractical, should provide teachers with many opportunities to help students develop a critical attitude towards statistically presented information. The legislation requires the media to report details about the polls that in the past were often absent. The size of the sample, the identity of those who commissioned the study, the results, the actual wording of any questions posed to the sample, and the sources of error will have to be presented, if the legislation passes. Teachers should demonstrate how some of these polling characteristics such as the wording of a question can influence the responses of people and, consequently, influence the results.

Students also need to learn to be wary of claims and arguments based on only partially presented or incomplete information. This tactic is often used by advertisers to misrepresent the virtues of their products. For example, TV viewers are exposed regularly to the claim that "90% of dentists recommend toothpaste X to reduce tooth decay." However, the critical viewer recognizes the incompleteness of the information. We are not told, for example, how many dentists were surveyed, nor do we know if other toothpaste brands were recommended too. Perhaps 90% of dentists recommend all common brands because there is no real difference between them in preventing tooth decay.

Another example is presented by Swift (1983). A TV commercial quoting a marketing survey on two brands of razors stated that 90% of the people who expressed a choice thought that brand A was better than, or no different from, brand X: a seemingly powerful argument for purchasing brand A. Of course, what had been withheld from the viewer is the breakdown of the 90 percent. Without this information, almost any claim can be made. For example, it is possible that 90% thought that there was no difference between the two brands and the remaining 10% favoured brand X and not brand A. Swift reported that only about 14% of grade 12 British Columbia students recognized the weakness of the advertiser's claim. (I suspect that the majority of educated adults would have similar difficulties in identifying the dubious nature of the claim.)

As has been indicated in both Parts 2 and 3 of this report, students and adults alike find information expressed as a percentage increase or decrease especially troublesome and confusing. When the base of a percentage statistic is not clearly identified, as often happens in department store flyers advertising a reduction to already discounted goods, most consumers believe they receive more savings than they actually do. For example, most adults assume mistakenly that a discount of 25%

followed by a further reduction of 25% results in a total saving of 50%.

When a base amount is successively increased or decreased by a percentage as in compound interest calculations, even educated adults demonstrate a remarkable lack of understanding. A representative of a large Saskatchewan employer recently tried to convince a labour arbitrator of the generosity of its wage offer by presenting him with the evidence that the employees enjoyed a substantial average annual wage increase over a period of several years. Much to the embarrassment of the employer and to the ultimate enjoyment of the employees, the statistic was all "smoke and mirrors" because it was based on an assumption of simple rather than on compound annual interest increases.

Students need to learn about elementary statistical terms such as average, variation, and percentile and covariation: their measurement, uses and properties. There is no need to involve students in elaborate calculations as the focus should be on the interpretation rather than the calculation of statistical measures. When calculation is necessary to help develop a statistical notion, simple numbers should be used; when handling relatively awkward real-world data, calculators should be used.

The two measures of central tendency, mean and median, are seen almost daily, yet few people are aware of their differing properties. When analysing data that is distributed relatively symmetrically such as the human characteristics of height, weight, foot size, etc., one kind of average is as good as another. However, in describing the average of skewed (asymmetrical) distributions, the type of average used must be known for the purposes of critical analysis. For example, suppose a company, wishing to promote the image that it is owned primarily by small stockholders, reports that 2000 people including 10 directors hold stock in the company and their average holding is 5000 shares. If the mean is used to express the average, it is possible that the 10 directors hold around 99 000 shares each, with the remaining 1990 shareholders holding around 50 shares each. Students should understand when it is appropriate to use a particular kind of average and the circumstances under which it might be misused deliberately to support the arguments of those with an axe to grind.

Students should be provided with at least a rudimentary understanding of covariation. In the senior secondary mathematics program, correlation techniques should be studied after an understanding of linear functions has been developed. The analysis of real-world linear relationships can be done quickly and painlessly by using inexpensive statistical calculators to calculate correlations. The teacher's responsibility should be to help students understand when it is appropriate to use such a statistical measure and how to interpret the meaning of a calculated coefficient.

In the other subject areas, the study of covariation need not be technical. Students must understand that if a correlation between two measures has been established, it does not necessarily imply that a cause/effect relationship exists. They should learn to question conclusions based entirely on an observed covariation and to consider alternative explanations of the resulting relationship. For example, the cost of a university education is commonly justified by the figures that show those with a degree ultimately earn more than those without one. The implication being that students would be better off financially by spending four years attending university than they would if these years were spent in doing something else. Of course, this conclusion may be unwarranted. Perhaps those who attend university will earn more than the general population even if they did not attend university. Universities still attract a disproportionate number of the brightest and most affluent young people, the ones most likely to succeed financially. If students are to learn to think critically, they must be taught to identify possibly unwarranted conclusions based on an observed correlation and be encouraged to provide alternative explanations.

Reasoning with Probability

The study of elementary probability should emphasise its use in estimating risks, chances, odds, and proportions. Real-world examples of the applications of probability abound and should be examined in the context of all school subjects, wherever possible. Students should see how probability is used in predicting genetic characteristics, calculating drug efficacy, estimating contraceptive failure, assessing various health risks and the chances of recovery from various illnesses, estimating risks to the environment from chemical pollutants and nuclear reactors, calculating life probabilities to be used in the construction of mortality tables, forecasting weather conditions, determining the chances of winning a lottery or other games of chance, etc.

The basic terms and definitions seen in the study of probability such as probability, event, outcome, chance, odds, frequency, combination, likelihood, etc. can be introduced in either the middle or junior high grades' mathematics program, after the students have a reasonable understanding of the fraction concept. At this stage, the instructional approach should be informal and activity-based and incorporate many hands-on activities and simple experiments relating empirical and theoretical probability. Students should collect, organize, tabulate, graph and interpret information based on experiments with dice throwing, coin flipping, thumbtack tossing, card drawing, etc. Students should learn to calculate the theoretical probability of an event by compiling an organized list of all possible outcomes. For example, in an experiment where two coins are flipped simultaneously, either an organized list of pairings or a simple tree diagram can be used to represent the possible outcomes of events and the various theoretical probabilities.

Once the possible outcomes of an event have been identified, questions about theoretical probabilities can be answered: e.g. (1) What is the probability that both coins will read heads? (2) What is the probability that heads appear on neither coins? (3) What is the chance that both a tail and a head face upwards? (4) If two coins were tossed 10 times, 100 times or 1000 times, about how many times would both a tail and a head be expected to appear? The lack of agreement between the expected frequencies and the observed frequencies resulting from the coin tossing experiment can serve as an illustration of the law of large numbers, which essentially states that small samples are more variable than larger samples.

In the secondary mathematics programs, more advanced notions of probability could be introduced, although a great deal of technical expertise is really not necessary to understand many of the fundamental notions of probability: for example, few people have either the knowledge or interest to calculate the chances of winning Lotto 649, but most can learn to interpret intelligently the expressions of probability associated with lotteries.

In all subject areas, an attempt should be made to help students understand commonly seen expressions of probability. For example, when the results of a political poll are reported by the media, the margin of error is usually reported as well, although few people including newspaper journalists and TV reporters appear to have a good understanding of this term. To increase students' understanding of this important technical idea, Social Studies' teachers should provide students with numerous examples of either incorrect or misleading statements that have been based supposedly on the findings of polls. For example, the Star-Phoenix reported recently the findings of the July Gallup poll that surveyed the political preferences of a sample of 1021 elegible voters. The newspaper reported that 41%, 35%, and 23% of decided voters would support the NDP, Liberals, and Conservatives, respectively, if a federal election was called immediately. It was also reported that this survey was considered accurate within four percentage points 19 out of 20 times. Despite their reporting of the margin of error, the newspaper claimed that the Tories "slipped" one percentage point while the Liberals "slumped" four percentage points from their standings in an earlier poll. When the margin of error of 4% is taken into account, however, neither term is justified because each drop in standing could reflect simply an error in measurement rather than a decrease in popularity.

Students need to understand how risks are calculated and how decisions to take risks are made. Many risk calculations are based on measurement and observation rather than on mathematical analyses. For example, the probability of being involved an automobile accident is based on traffic statistics collected over a period of time. Students need to learn to discriminate between probability calculations based on theoretical arguments, actual measurements or fancy guesswork.

Errors in the calculation of probability are necessarily more considerable when empirical probablities are derived from statistics.

Students should be taught to consider carefully the possibility of error when asked to judge someone elses' assessment about the risks associated with some venture, the building of a nuclear power plant, for example. Students should also understand that probability is a measure of uncertainty, a somewhat perplexing notion for most people, and that even highly improbable events can and do take place given enough time and enough instances.

Most of us knowingly take risks daily without a knowledge of the exact probabilities associated with those risks. We drive our cars knowing there is some chance of being killed or injured. We eat commercially-prepared food knowing that some ingredients are likely carcinogenic. Some of us take more risks than others by driving without a seatbelt, by smoking, by drinking, or by sky-diving. A knowledge of probabilities does not mean that people will necessarily make the best decisions: just as price is not the only consideration in making a purchase, probability alone must not dictate decisions in the matter of taking a risk. To help students become better decision makers, they must be encouraged to consider and weigh the possible benefits and consequences associated with the making of a decision and never to base a decision entirely on risk calculations alone.

Although Laplace, a famous 19th century mathematician, suggested that probability is common sense complemented by computation, common sense regarding some notions of probability is not that common. There are aspects of probability that students and adults find especially confusing and contrary to what is generally called common sense (see Hope & Kelly (1983)).

One commonly misunderstood idea about probability is often referred to as the "gambler's fallacy." People tend to assume that even a few instances of some random process must conform to a theoretical pattern. For example, the 1977 U.S. National Mathematics Assessment (NAEP) found that the great majority of 17-year-olds suffer from this form of faulty reasoning: over 70% responded incorrectly to the item "In three tosses of a fair coin, heads turned up twice and tails turned up once. What is the probability that heads will turn up on the fourth toss?" (p. 319). Only 30% of the students realized that the probability remained at one half, regardless of the past performances of the coin.

There are many real-world examples that illustrate how tempting it can be for people to succumb to the gambler's fallacy. For example, when a woman gives birth to a consecutive number of babies of one sex, there is a common belief that the sex of the next child born will be likely different. Perhaps Mrs. Emory Harrison, who gave birth to thirteen consecutive boys, was inspired to further motherhood by a mistaken belief that the next child "just has to be a girl." Baby gender like the turn of a roulette wheel, throw of the dice, and shuffling of cards has neither the memory of past events nor the decency to make up for the errors of the past.

Many people also seem to abandon their common sense when they purchase lottery tickets. They do seem to understand that fairly-run lotteries use a random process to select the winning numbers and, consequently, that all ticket numbers have an equal chance of being selected as winners. Yet many people who participate in those lotteries where ticket numbers can be chosen will not choose certain number combinations such as 123456, 555555, or 121212 because they mistakenly assume that these ticket numbers are so remarkable that they are less likely to be winners. This idea that "remarkable" events are more unlikely to occur than less remarkable events is reinforced when it is noted that no winning lottery numbers published by the media are particularly remarkable. The fact that less peculiar numbers did not win either is often ignored or goes unnoticed.

Students need to learn to analyze critically schemes purporting to increase the chances of winning a lottery. Recently the Star-Phoenix ran a full page ad (February 18, 1987, p. A13) in which a supposed "numbers genius" had developed a successful scheme for winning lotteries that she was willing to share with others for the meagre sum of $20.00. The ad contained a number of statements that teachers could use to challenge students to think critically. For example, the ad claimed that the scheme "increased one's chances of winning Canadian or U.S. Lotteries by more than 500%!" Undoubtedly, this figure is intended to impress the reader. However, it will have less impact on those people who realize that the chances of winning can be increased 500% simply by purchasing 5 tickets instead of 1 ticket.

When a peculiar event happens, people often underestimate its likelihood. For example, any five-card poker hand in a fair game is equally likely to be dealt out, but because some equally unlikely hands are considered preferable to others such as a royal flush, card players often attach greater significance to its rarity than is warranted. What seems not to be understood is that unpreferable hands like the 8 of spades, king of diamonds, ace of hearts, jack of clubs, and 4 of diamonds are as rare as any of the more valued hands.

All subject areas must help students understand these seemingly counter-intuitive ideas about probability. Students should also learn how others will try to take advantage of their ignorance of some fundamental notions of probability. For example, mystics, astrologers and other charlatans will often justify a prediction, usually for the purpose of unloading the bank accounts of the ignorant and gullible, on the supposed rarity of some event such as a particular alignment or arrangement of planets. These types of astrological predictions ignore the fact that one heavenly arrangement is as likely as any other arrangement.

Students should learn that highly sensational stories about coincident events can also be explained without resorting to supernatural explanations. For example, an American television program recently reported the "remarkable" story of a man who dreamt of a murder that happened the next day. Such a coincidence can be explained by considering the following possibilities: (1) It is highly probable that a large number of Americans over a period of years have several dreams involving violence, including murder; (2) Of these dreams about murders, many similarities between them and real murders will exist; (3) A few dreams will be so similar to actual murders that they will be given an unusual amount of publicity. Students should understand that in large populations it is almost certain that someone at sometime will experience a "remarkable coincidental experience."

There are a number of publications that can provide curriculum developers and teachers with instances of the real-world uses of probability and statistics. Presenting secondary students with some of these applications can show them how probability and statistics can be used either to illuminate or confound:

1. Statistics: A Guide to the Unknown published by Holden-Day contains many useable and interesting essays written by experts in the

field. Essays such as the following can be understood without an extensive knowledge about statistics and probability: 1) "Do speed limits reduce traffic accidents?" 2) "Opinion polling in a democracy." 3) "Statistics, sports, and some other things."

2. Statistics Made Relevant: A Casebook of Real Life Examples published by John Wiley and Sons contains many analyses of cases largely drawn from the news media. Although the book was designed as a supplement to a more formal study of statistics, many of the cases require little technical knowledge while others can be modified to eliminate possibly confusing "number-crunching."

3. Teaching Statistics and Probability was written primarily for elementary and secondary mathematics teachers and prepared and published by the National Council of Teachers of Mathematics. However, there are articles that would be useful for other teachers as well: for example, one article written by an employee of Statistics Canada shows how a Student Price Index created along the lines of the CPI can help students conceptualize in simple terms the movement of prices, the workings of the market, and the nature of a nation's economy.

4. Statistics by Example is a series of booklets prepared by the Joint Committee on the Curriculum in Statistics and Probability of the American Statistical Association and the National Council of Teachers of Mathematics and published by Addison-Wesley that provide examples of statistical activity seen in the media, science and industry. Because they were designed primarily with secondary mathematics courses in mind, most of the examples are best studied in a senior mathematics course.

5. How to Lie with Statistics by Darrell Huff, mentioned earlier in this paper and listed in the references, is a book that is easily

understood by those claiming not to have a mathematical mind. It provide

many examples to illustrate the use and misuse of statistics.

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APPENDIX 1

A LIST OF UNDERSTANDINGS AND COMPETENCIES NEEDED BY NUMERATE PEOPLE TO HANDLE THE DEMANDS OF FUTURE ORDINARY LIFE

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1. Knows How To Use Mathematics To Solve Problems

Understands how mathematical concepts, rules and procedures can be applied to everyday situations and how to make use of them to carry out familiar tasks and solve unfamiliar problems.

Approaches non-routine problems with confidence and is willing to tolerate the inevitable frustration while pursuing a solution.

> Recognizes situations where adding, subtracting, multiplying, dividing or some combination of these processes is involved with special emphasis on the handling of common money and measurement problems.
>
> Recognizes situations where ratios and proportions can be applied with special emphasis on handling percentages, determining the better buy of a purchase, converting measurement units, interpreting scales, and carrying out other types of practical tasks.
>
> Demonstrates an understanding of a problem by providing an alternative rephrasing.
>
> Recognizes when problem statements require additional information and knows how to acquire this information.
>
> Identifies extraneous information included in a problem statement.
>
> Reduces a complex problem into a sequence of more manageable subproblems.
>
> Organizes information into a list, table, graph or chart and uses it to look for and generalize from a numerical or geometric pattern.
>
> Guesses a solution, checks its suitability by examining the conditions of the problem, and refines the guess accordingly.
>
> Draws a diagram or makes a model to represent a problem situation.
>
> Applies a logical principle in the solution of a problem.
>
> Relates to an analogous but simpler problem.
>
> Determines the reasonableness of an answer to a problem by using a computational estimation technique, by comparing the answer to a known referent, or by using knowledge gained from experiences in handling similar problems.

Verifies answers by referring to the problem requirements, by checking the validity of each step of the method of solution, by looking for errors in computation, reasoning or information and, wherever appropriate, by using an alternative method of solution.

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2. Knows How To Perform Calculations

Carries out calculations involving whole and decimal numbers, fractions and percentages using variety of modes and methods including the use of mental methods, paper-and-pencil methods, calculators, and ready reckoners and knows when it is appropriate to use a particular mode or method.

Determines the size of quantities by using some form of a counting procedure.

Possesses confident recall of addition and multiplication number combinations to 9 + 9 and 9 x 9 and the related subtraction and division facts.

Uses some easily understood pencil-and-paper methods for carrying out commonly done whole and decimal number calculations, especially for handling money and measurement tasks.

Can use a wide variety of mental methods for determining the sum, difference, product and quotient of straightforward numbers and uses these methods to form a reasonable estimate.

Distinguishes between those situations where a calculation should involve exact or approximate numbers.

Knows how and when to simplify a calculation.

Explains why a computational procedure works by identifying the underlying number principles.

Add and subtracts fractions with straightforward and commonly seen denominators as 2, 4, 8, 16, 10 and 100.

Multiplies and divides simple fractions.

Calculates with ratios, proportions and works with percents.

Knows how and when to change a decimal, whole or fraction to another equivalent number, or a close approximation.

Knows by heart the equivalences between commonly seen decimals, fractions, and percentages and can use them to simplify a calculation, solve a problem, or produce an estimate.

Knows a number of equivalent procedures for calculating with whole and decimal numbers, fractions and percents and can recognize situations where one procedure is preferable to another.

Uses a variety of ready reckoners to figure out calculations involving interest payments, sales tax, SI/Imperial conversions, etc.

Uses a calculator efficiently to add, subtract, multiply and divide whole and decimal numbers.

Knows how and when to use the following special function keys on a calculator: decimal, memory, correct error, percent, clear memory, clear display, change sign, square root.

Reads and enters exponential information on a scientific calculator.

Operates a SI/Imperial conversion calculator and is familiar with some of the functions on other types of special-purpose calculators.

Selects from the calculator display the number of figures appropriate to the context of the calculation.

Appreciates the need for careful ordering of operations when using a calculator.

Checks the accuracy of a calculation by repeating all steps in a different order using the same operation or by using a different operation.

Knows the limitations of the calculator and when it is important to take these limitations into account.

Substitutes numbers in a simple formula expressed in words and evaluates the answer.

3. Knows How To Measure

Develops an awareness and appreciation of measurement and its use in everyday life.

Understands that all measurement involves the quantification of properties or states.

Can use a wide range of devices calibrated in either SI or Imperial units to measure length, area, volume, capacity, temperature, mass, time and angles.

Reads commonly seen dials, meters, and scales and understands how to interpret these readings.

Tells the time using both the digital and traditional clockface and understands both 12- and 24-hour notation.

Recognizes situations where measurement is necessary and selects the appropriate measuring tools.

Reads and understands expressions of measurement including those where the measurement units are not clearly identified and must be determined from the context.

Knows the names of the common SI units and their symbols for length, area, volume, capacity, temperature, mass and time.

Knows the commonly seen prefixes, their symbols and their decimal values and how they can be used in conjunction with SI units.

Understands the decimal structure of the SI measurement system and can use this understanding to convert commonly seen measurements to an equivalent measurement involving a larger or smaller unit with special emphasis on the following: millimetres, centimetres, metres, kilometres; grams, kilograms, tonnes; millilitres, centilitres, litres.

Knows and uses the following SI equivalences and can use them to solve problems or assess the reasonableness of a measurement expression: 1 litre is equivalent to 1000 cubic centimetres, 1 millilitre is equivalent to 1 cubic centimetre; 1 litre of water has an approximate mass of 1 kilogram, 1 millilitre of water has an approximate mass of 1 gram, 1 cubic metre of water has an approximate mass of 1 tonne.

Demonstrates a "feel" for the size of the common SI units by associating each unit to a concrete referent or situation and uses these associations to assess the reasonableness of a measurement calculation or quantitative expression.

Knows when and how to make decisions based on visual inspection rather than on accurate measuring.

Understands the meaning of precision and determines the most appropriate degree of precision for a needed measurement.

Knows how to calculate the area of simple figures and the volume of common solids and can use this knowledge to carry out practical tasks.

Understands common expressions containing compound units as seen in rates and ratios.

Knows how to use Imperial units and can provide their approximate SI equivalents with special emphasis on the following units and equivalences: inch, foot, yard, mile, acre, section, pound, quart, gallon; 3 feet is about a metre, 5 miles is about 8 kilometres, 1 hectare is about 2 1/2 acres, 2 pounds is about 1 kilogram, 1 gallon is about 4 1/2 litres.

Uses tables of conversion factors, tables of equivalences or an SI/Imperial calculator to convert from one measurement system to another when necessary.

4. Knows How To Work With Space And Shape

Understands the fundamental properties of space and of commonly seen geometric figures.

Recognizes and names simple plane figures and uses the associated geometric terms with understanding: centre, side, radius, diameter, diagonol, circumference, angle and perimeter.

Recognizes and names common solid shapes and uses the associated geometric terms with understanding: face, edge, and vertex.

Draws a simple plane figure to given specifications using common drawing instruments including ruler, compass, set square and protractor.

Recognizes the two-dimensional drawing corresponding to a given solid and can use these drawings for building as needed.

Appreciates the concept of scale in geometric drawings and maps, and the use of co-ordinates to locate areas and points.

Carries out a practical task by applying knowledge of geometric properties of simple shapes.

5. Knows How To Analyze And Interpret Quantitative Information

Develops an inquiring attitude about the veracity and of statistical information and its use in supporting an argument or position.

Reads, writes and orders positive and negative whole numbers and decimal numbers and uses them in context.

Uses an understanding of number to read and complete forms.

Understands the conventions and terminology of the decimal numeration system.

Understands the need to use an appropriate level of precision when using expressions of quantity.

Reads and extracts information presented in tabular form and organizes systematically the collection of simple data.

Reads, interprets, and extracts information from commonly seen graphs and charts and, in simple cases, constructs them.

Reads and extracts information from commonly seen timetables and the calendar.

Understands the meaning and uses of quantitative terms commonly used by business, media and government.

Knows the meaning of the term margin of error and understands how error must be considered when examining the results of polls and surveys.

Understands how statistics can be used to support an argument or claim and is aware that the same statistical figures can lead to seemingly different conclusions.

Has an awareness of the reporting techniques commonly used by special interest groups to increase the impact of data and influence the uncritical reader or listener.

Understands the idea of population sampling and its use in surveys, polls, and empirical studies.

Understands the fundamentals of probability and their use in expressing risks and chances, estimating proportions in a population and making predictions.

Knows how to calculate simple probabilities and odds of single and joint events.

Knows the meaning of the following terms: probability, chance, likelihood, event, odd, frequency, combination.

Recognizes when an expression of probability has been determined theoretically or empirically.

Identifies the assumptions underlying the calculation of a statistic or probability, recognizes when an assumption is violated, and understands the consequences of a violation.

Understands the meaning of average and variability, the various ways these statistics are measured, and the purposes for which they are used.

Reads and uses percentages seen in daily life and examines critically arguments based on percentage increase or decrease.

APPENDIX 2

RECOMMENDATIONS

The recommendations that follow are based on the research findings and arguments presented in the main body of this paper. It is recommended that:

1. The promotion of numeracy be considered an important goal of all school subjects and that curriculum developers and teachers integrate the understandings and competencies identified in this paper into each subject area and design appropriate learning activities based on the examples and discussion provided in this paper.

2. The mathematics program be revised to ensure that the development of numeracy becomes the principal goal of the elementary, middle and secondary mathematics programs.

2.1 Special emphasis be given to concept development, organizing and interpreting data, using mathematical ideas to carry out tasks and solve problems commonly seen outside of school, developing a number sense, acquiring a feel for measurement, calculating with technological aids, interpreting and assessing the reasonableness of calculations, and analysing critically arguments and conclusions based on quantitative information.

2.2 Computing with numbers that are either unrealistic or stripped of a meaningful context be avoided. Generally speaking, mathematical procedures and algorithms should always be introduced and practised in the context of handling commonly seen tasks or solving realistic problems.

2.3 A greater emphasis be placed on calculating mentally and producing estimates of quantities.

2.4 The fundamentals of statistics and probability and their common uses be studied, beginning in the middle grades and continuing throughout all secondary grades.

2.5 The secondary mathematics program be modified to incorporate the advanced notions of arithmetic that most students and adults find difficult to understand, especially the handling of percentages, decimals and fractions seen in everyday life.

2.6 The informal methods used by adults to handle common everyday tasks and solve routine problems be included for study.

2.7 Students in all grades be provided regularly with practical experiences involving the use of measurement devices, calculating aids, construction tools and other types of manipulative materials. Schools should be provided with a list of recommended materials as well as suggestions for their use.

2.8 The types of problems presented in school not be limited to conventional word problems whose solution requires merely the application of a previously learned algorithm. More problems requiring higher-level thinking processes should be presented in all school subjects.

3. Saskatchewan Education develop and publicize a policy supporting the use of calculators in all grades and in all school subjects.

3.1 All students learn to use a variety of different types of calculators in a confident, efficient and intelligent manner to handle the arithmetic needed in carrying out practical tasks and in solving realistic problems.

3.2 All students have access to calculators whenever a calculation performed by hand either takes up an unwarranted amount of instructional time or obscures the mathematical process under study.

3.3 Calculators be used in school to develop mathematical ideas and to explore number patterns as well as to carry out calculational tasks.

3.4 Calculators be permitted on tests and examinations except when the ability to carry out straightforward calculations done with paper and pencil or in the head is being evaluated.

4. Saskatchewan Education or a related government agency sponsor a study to determine if a knowledge of Imperial measurement is still required by employers and if a lack of knowledge by young workers is a cause for concern. The study should also determine whether SI measurement is used exclusively in all subject areas by all teachers.